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A SIMPLE STATISTICAL MODEL OF DAILY GLOBAL SOLAR RADIATION IN THAILAND

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Abstract

Three simple formulas are given that represent respectively the daily global solar radiation under a clear sky as a function of latitude and day of the year, the expectation value of the daily clearness factor as a function of the day of the year at specified locations, and the distribution of daily clearness factors as a function of the expectation value.

Three years ago a mathematical model for solar radiation in Thailand was constructed and programmed for use on a large computer.[1,2] The model was complicated and contained many parameters. In order to make the model more accessible to users it has recently been simplified. This paper gives details of the new method of simulating the fluctuating day-to-day totals of global solar radiation. The equations used can be evaluated more quickly, and contain fewer parameters, than those in the old model ; the resulting loss in accuracy is insignificant in view of the variability of solar radiation from year to year and the limitations in the data upon which the models are based.

The new model consists essentially of three basic functions. The first represents the daily total global solar radiation H_c from a clear sky in terms of geographic latitude ϕ north of the equator and the day of the year n counting from n = 1 on 1 January to n = 365 on 31 December. The function is :

$$H_c = F(A_0 + A_1 \cos(t - 92) + A_2\cos(2t - 4)), \quad (1)$$

where $t = (n - 80)\,360/365$ degrees,

$A_0 = 27.420 + 0.144x - 1.702x^2$ MJ/m^2,

$A_1 = 0.056 + 6.308x + 0.064x^2$ MJ/m^2,

$A_2 = 1.211 - 0.041x - 0.099x^2$ MJ/m^2,

$x = \phi\ 20$ with ϕ in degrees,

and $F = 1 - 0.0335\sin(n - 94)\,360/365$.

This formula reproduces the values given by Schüepp[3] within 1%.

The philosophy underlying equation (1) is that annual variations of climatic elements, such as daily solar radiation, should be represented by periodic functions of length one year ; the March equinox (21 March) is a convenient natural origin for the time scale. This replaces the use of calendar months, which are artificial subdivisions of the year. The factor F represents the effect of the variation of the earth-sun distance.

Next let H denote the daily total global solar radiation on a particular day, and define a daily clearness factor k by the equation

$$H = kH_c.$$

The second basic function of the model represents the expected value E(k) of k in terms of the day of the year thus :

$$E(k) = C_0 + C_1\cos(t - \phi_1) + C_2\cos(2t - \phi, \quad (2)$$

where t is the same as before and the parameters C_0, C_1, ϕ C_2 and ϕ_2 must be determined from measurements or estimates of the annual variation of insolation at each geographical location of interest. Table 1 shows the parameters calculated from estimates of daily solar global solar radiation at four stations using mean values of k in eight $1\frac{1}{2}$ month periods of the year and standard methods of numerical harmonic analysis.[4]

The third basic function in the model represents the probability distribution P(k) of k, restricted to the discrete set k = 0.05, 0.15, ... , 0.85, 0.95, by the equation :

$$B(k) = (9\,!\,/\,x\,!\,(9 - x)\,!\,)^x(1 - p)^{9-x}, \quad (3)$$

where $p = (10E(k) - 0.5)/9$

and $x = 10k - 0.5$.

This formula slightly overestimates the dispersion of the distribution when E(k) is large, and slightly underestimates it when E(k) is small but the error is not serious. A better formula would be the beta distribution, which is continuous over the domain k = 0 to k = 1 and can be adjusted to represent correctly both the mean and the standard deviation of k. Unfortunately the computational effort needed to evaluate beta distributions makes them unsuitable for simple mathematical models.

The information contained in equations (1), (2) and (3) can be used in various ways. For example, if the daily output of a system utilizing solar energy is known as a function of the daily total global solar radiation, the equations enable one to estimate the distribution of outputs throughout the year. Alternatively, one can use the equations in conjunction with a random number generator to produce fluctuating sequences of daily solar radiation values that resemble actual sequences. Persistence effects, though significant, are slight in the climate of Thailand, so not much accuracy is lost by neglecting them.[5]

The model can be extended to represent hourly values of global, direct and diffuse solar radiation. For this purpose the original model may be used on account of its simplicity, except that differences between the morning and afternoon distributions of solar radiation are not sufficiently marked to justify incorporation into a simplified model. The basic formulas required are available elsewhere.[2]

TABLE 1. PARAMETERS FOR EQUATION (1) AT FOUR STATIONS

Station	C_2	C_1	ϕ_1	C_2	ϕ_2
Chiang Mai	0.681	0.112	293°	0.040	94°
Khon Kaen	0.669	0.100	286°	0.044	137°
Bangkok	0.636	0.099	302°	0.023	139°
Songkhla	0.616	0.050	1°	290°	

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บทคัดย่อ

ได้เสนอสูตรง่าย ๆ 3 สูตร สำหรับคำนวณการแผ่รังสีประจำวันจากดวงอาทิตย์สู่โลก เมื่อท้องฟ้ากระจ่างเป็นฟังชั่นของเส้นรุ้งและวันในปี ค่าที่คาดหวัง (expectation value) ของความแจ่มใสประจำวัน (daily clearness factor) เป็นฟังก์ชั่นของวันในปีในสถานที่ที่กำหนดและการกระจายของค่าความแจ่มใสประจำวันเป็นฟังก์ชั่นของค่าที่คาดหวัง