RESEARCH ARTICLE
doi: 10.2306/scienceasia1513-1874.2016.42.022
ScienceAsia 42 (2016): **22–27**

Nuclear structure of yrast bands of ^{180}Hf, ^{182}W, and ^{184}Os nuclei by means of interacting boson model-1

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Received 30 Jul 2015
Accepted 30 Nov 2015

ABSTRACT: In this paper, an interacting boson model (IBM-1) has been used to calculate the low-lying positive parity yrast bands in Hf, W, and Os nuclei for $N = 108$ neutrons. The systematic yrast level, electric reduced transition probabilities $B(\text{E2})\downarrow$, deformation, and quadrupole moments of those nuclei are calculated and compared with the available experimental values. The ratio of the excitation energies of first 4^+ and first 2^+ excited states, $R_{4/2}$, is also studied for these nuclei. Furthermore, as a measure to quantify the evolution, we have studied systematically the yrast level $R = (\text{E2} : L^+ \rightarrow (L-2)^+)/(\text{E2} : 2^+ \rightarrow 0^+)$ of some low-lying quadrupole collective states in comparison to the available experimental data. The associated quadrupole moments and deformation parameters have also been calculated. Moreover, we have studied the systematic $B(\text{E2})$ values, intrinsic quadrupole moments, and deformation parameters in those nuclei. The moment of inertia as a function of the square of the rotational energy for even atomic numbers $Z = 72$, 74, 76 and $N = 108$ nuclei indicates the nature of the back-bending properties. The results of these calculations are in good agreement with the corresponding available experimental data. The analytic IBM-1 calculation of yrast levels and $B(\text{E2})$ values of even-even Hf, W, and Os for $N = 108$ nuclei were performed in the SU(3) character.

KEYWORDS: energy level, reduced transition probabilities, quadrupole moments, deformation parameter

INTRODUCTION

The interacting boson model-1 (IBM-1) developed by Iachello and Arima[1–3] has been successful in describing the collective nuclear structure for the prediction of the low-lying states and the electromagnetic transition rates in the medium mass nuclei. The IBM-1 has become one of the most intensively used nuclear models, due to its ability to describe of the changing low-lying collective properties of nuclei across an entire major shell with a simple Hamiltonian. In first approximation, only pairs with angular momentum $L = 0$ (called s-bosons) and $L = 2$ (called d-bosons) are considered. The model has associated an inherent group structure, which allows for the introduction of limiting symmetries called U(5), SU(3), and O(6)[4,5].

The nuclei ^{180}Hf, ^{182}W, and ^{184}Os, have atomic number $Z = 72$, 74, and 76, respectively, and same neutron number $N = 108$ are existed on the stability line. Those nuclei are very much of interest because their balance nucleons are proton-neutron holes according to double shell closure ^{208}Pb and are well deformed. It is known that the low-lying collective quadrupole E2 excitations occur in even-even those nuclei, which have been studied both theoretically and experimentally[6–9].

There are a number of theoretical works discussing intruder configuration and configuration mixing by means of IBM-1 around the shell closure $Z = 82$. For instance, empirical spectroscopic study within the configuration mixing calculation in IBM-1[10], the IBM-1 configuration mixing model in strong connection with shell model[11], conventional collective Hamiltonian approach[12] and the one starting from self-consistent mean-field calculation with microscopic energy density functional.

Recently we studied the evolution properties of the yrast states for even-even $^{100-110}$Pd isotopes[13]. The yrast states and electromagnetic reduced transition probabilities of even-even $^{114-122}$Cd isotopes were studied by Hossain et al[14]. U(5) symmetry of even ^{110}Pd, ^{110}Cd, and $^{96-100}$Ru isotopes were studied within the framework of the IBM[15,16]. Electromagnetic reduced transition properties of yrast states band of even-even $^{102\text{–}112}$Pd isotopes were studied[17,18]. Previous studies motivate the aim of the present work by application of IBM-1 to predict the yrast level, reduced transition probabilities and back bending curve to understand the type of dynamical symmetry which exist in Hf, W, and Os nuclei for neutron $N = 108$.

THEORETICAL CALCULATIONS

Interacting boson model (IBM-1)

The interaction of s-bosons ($L = 0$) and d-bosons ($L = 2$) in the IBM are used to explain the collective properties of even-even nuclei[19].

The IBM-1 Hamiltonian can be expressed as[3]:

$$\begin{aligned} H &= \varepsilon_s(s^\dagger \cdot \tilde{s}) + \varepsilon_d(d^\dagger \cdot \tilde{d}) \\ &\quad + \sum_{L=0,2,4} \tfrac{1}{2}(2L+1)^{1/2} C_L \big[D_{11}^{(L)} \times D_{22}^{(L)}\big]^{(0)} \\ &\quad + \tfrac{1}{\sqrt{2}} v_2 \big[D_{11}^{(2)} \times C_{22}^{(2)} + C_{11}^{(2)} \times D_{22}^{(2)}\big]^{(0)} \\ &\quad + \tfrac{1}{2} v_0 \big[D_{11}^{(0)} \times S_{22}^{(0)} + S_{11}^{(0)} \times D_{22}^{(0)}\big]^{(0)} \\ &\quad + \tfrac{1}{2} u_0 \big[S_{11}^{(0)} \times S_{22}^{(0)}\big]^{(0)} + u_2 \big[C_{11}^{(2)} \times C_{22}^{(2)}\big]^{(0)}, \quad (1) \end{aligned}$$

where

$$\begin{aligned} D_{11} &= d^\dagger \times d^\dagger, & D_{22} &= \tilde{d} \times \tilde{d}, \\ C_{11} &= d^\dagger \times s^\dagger, & C_{22} &= \tilde{d} \times \tilde{s}, \\ S_{11} &= s^\dagger \times s^\dagger, & S_{22} &= \tilde{s} \times \tilde{s}. \end{aligned}$$

This Hamiltonian contains 2 terms of one body interactions (ε_s and ε_d), and 7 terms of two-body interactions [c_L ($L = 0, 2, 4$), v_L ($L = 0, 2$), u_L ($L = 0, 2$)], where ε_s and ε_d are the single-boson energies, and c_L, v_L, and u_L describe the two-boson interactions. However, it turns out that for a fixed boson number N, only one of the one-body terms and five of the two body are terms independent, as it can be seen by noticing that $N = n_s + n_d$. Then the IBM-1 Hamiltonian in (1) can be written in a general form as[4,5]

$$\begin{aligned} \hat{H} = \varepsilon \hat{n}_d + a_0 \hat{P} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} \\ + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4, \quad (2) \end{aligned}$$

where $\hat{n}_d = (d^\dagger \cdot \tilde{d})$ is the total number of d_{boson} operator, $\hat{P} = \frac{1}{2}[(\tilde{d} \cdot \tilde{d}) - (\tilde{s} \cdot \tilde{s})]$ is the pairing operator, $\hat{L} = \sqrt{10}[d^\dagger \times \tilde{d}]^{(1)}$ is the angular momentum operator, $\hat{Q} = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + \chi [d^\dagger \times \tilde{d}]^{(2)}$ is the quadrupole operator (χ is the quadrupole structure parameter and takes the value 0 in the case of O(6) symmetry and $\pm\sqrt{7}/2$ corresponding to SU(3)), $\hat{T}_r = [d^\dagger \times \tilde{d}]^{(r)}$ is the octupole ($r = 3$) and hexadecapole ($r = 4$) operator, and $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy. The parameters a_0, a_1, a_2, a_3, and a_4 designated the strength of the pairing, angular momentum, quadrupole, octupole, and hexadecapole interaction between the bosons.

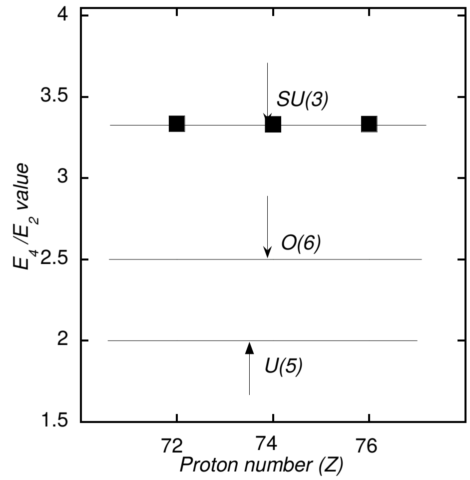

Fig. 1 $E(4_1^+)/E(2_1^+)$ value as a function of atomic number (Z) in ^{180}Hf, ^{182}W, and ^{184}Os nuclei.

RESULTS AND DISCUSSION

In Hf, W, and Os nuclei with neutron $N = 108$, proton hole numbers are 5, 4, and 3 and neutron hole number is 9 according to framework of IBM-1. The total boson numbers are 14, 13, and 12 for ^{180}Hf, ^{182}W, and ^{184}Os nuclei, respectively. The symmetry shape of a nucleus can be predicted from the energy ratio $R = E(4_1^+)/E(2_1^+)$, where $E(4_1^+)$ is the energy level at 4_1^+ and $E(2_1^+)$ is the energy level at 2_1^+. Actually R has a limit value of ≈ 2 for the vibration nuclei U(5), ≈ 2.5 for γ-unstable nuclei O(6) and ≈ 3.33 for rotational nuclei SU(3). The R values of low-lying energy levels of ^{180}Hf, ^{182}W, and ^{184}Os nuclei are 3.33, 3.33, 3.33 and the experimental values are 3.31, 3.29, 3.20, respectively, which are shown in Fig. 1. From Fig. 1, we have predicted SU(3) symmetry in even-even ^{180}Hf, ^{182}W, and ^{184}Os nuclei.

Yrast levels

The yrast levels (2, 4, 6, 8, ..., 14) of ^{180}Hf, ^{182}W, and ^{184}Os nuclei were calculated by taking the number of free parameters in the Hamiltonian to a minimum. These parameters are determined from the experimental energy levels (2^+ and 4^+).

Table 1 The parameters used in the IBM-1 calculations.

Nucl.	$N_\pi + N_\nu = N$	a_1 (MeV)	a_2 (MeV)	χ
^{180}Hf	$5+9=14$	0.0210	0.0268	−1.33
^{182}W	$4+9=13$	0.0109	−0.0154	−1.32
^{184}Os	$3+9=12$	0.0156	−0.0117	−1.32

Table 2 Comparison of theoretical and experimental excitation energies (MeV) of ^{180}Hf, ^{182}W, and ^{184}Os nuclei.

Nucl.	IBM-1			Experimental		
	$E(2)$	$E(4)$	$E(4)/E(2)$	$E(2)$	$E(4)$	$E(4)/E(2)$
^{180}Hf	0.093	0.311	3.335	0.093	0.309	3.31
^{182}W	0.100	0.334	3.332	0.100	0.329	3.29
^{184}Os	0.120	0.399	3.333	0.120	0.384	3.20

Each nucleus at the evolving states is determined using (2). Table 1 shows the IBM-1 parameters that are used in the calculations of yrast states of those nuclei. In the calculations the value of ε, a_0, a_3, a_4 are taken as zero value. Table 2 shows comparisons of theoretical and experimental excitation energies (in units of MeV) up to the first 4^+ levels and their ratio $R = E(4_1^+)/E(2_1^+)$ gives the energy level fit as well as rotational and gamma soft nuclear deformation.

Fig. 2 shows the comparison of the ratios $R_L = E(L^+)/E(2_1^+)$ as a function of angular momentum (L) in the yrast band for those nuclei. To measure the evolution of nuclear collectively, we present energies of the yrast sequences using IBM-1 (normalized to the energy of their respective 2_1^+ levels) in those nuclei and have compared with previous experimental values[20–22]. From the figure ^{180}Hf, ^{182}W, and ^{184}Os, we can see that IBM-1 calculation fit the SU(3) character. However, the comparison between the calculations and the experimental values are excellent and R_L increased towards higher spin state. The R_L values of ^{180}Hf, ^{182}W, and ^{184}Os indicate that excitation of those nuclei are similar as those are same SU(3).

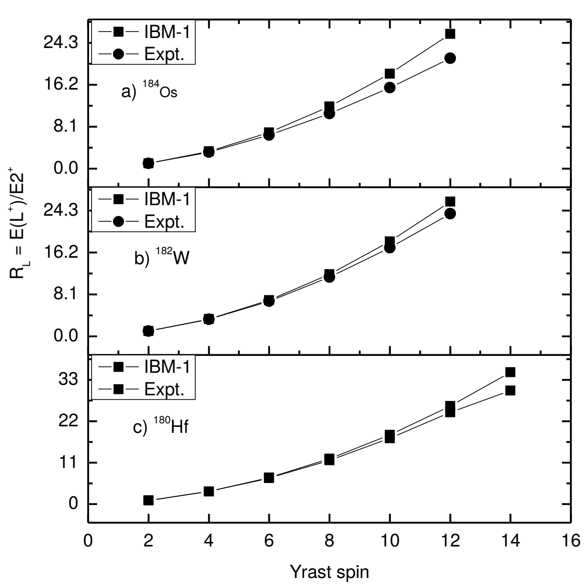

Fig. 2 $R_L = E(L^+)/E(2_1^+)$ as a function of angular momentum (L) in the yrast band for (a) ^{180}Hf, (b) ^{182}W, and (c) ^{184}Os nuclei.

Table 3 Reduced transition probability $B(\text{E2})\downarrow$ in even ^{180}Hf, ^{182}W, and ^{184}Os nuclei.

Nucl.	α (W.u.)	Yrast level	Energy (keV)	Transition level	$B(\text{E2})_{\text{exp}}$ (W.u.)	$B(\text{E2})_{\text{cal}}$ (W.u.)
^{180}Hf	1.34(0.24)	2	93.3	$2^+ \to 0^+$	155(5)	155.63
		4	308.6	$4^+ \to 2^+$	230(30)	220.08
		6	640.9	$6^+ \to 4^+$	219(22)	237.32
		8	1083.9	$8^+ \to 6^+$	250(40)	240.74
		10	1630.4	$10^+ \to 8^+$	240(13)	236.94
		12	2272.4	$12^+ \to 10^+$	232(10)	228.16
^{182}W	1.34(0.05)	2	100.1	$2^+ \to 0^+$	136.1(1.8)	136.1
		4	329.4	$4^+ \to 2^+$	196(10)	191.85
		6	680.4	$6^+ \to 4^+$	201(22)	206.19
		8	1144.3	$8^+ \to 6^+$	209(18)	208.11
		10	1711.9	$10^+ \to 8^+$	203(19)	203.37
		12	2372.6	$12^+ \to 10^+$	191(10)	193.92
^{184}Os	1.24(0.15)	2	119.8	$2^+ \to 0^+$	99.6(1.5)	99.6
		4	383.9	$4^+ \to 2^+$	140(40)	140.09
		6	774.1	$6^+ \to 4^+$	> 0.44	149.94
		8	1274.8	$8^+ \to 6^+$	> 0.13	150.37
		10	1871.2	$10^+ \to 8^+$	> 0.054	145.61
		12	2547.6	$12^+ \to 10^+$	> 0.024	137.08

$B(\text{E2})_{\text{exp}}$: Refs. 20–22; $B(\text{E2})_{\text{cal}}$: IBM-1 calculations.

Reduced transition probabilities *B*(E2)

The low-lying levels of even-even nuclei ($L_\text{i} = 2$, 4, 6, 8, ...) usually decay by one E2 transition to the lower-lying yrast level with $L_\text{f} = L_\text{i} - 2$. The reduced transition probabilities in IBM-1 are given for the harmonic vibration limit SU(3)[23]:

$$B(\text{E2}; L+2 \to L)\downarrow = \frac{3(2N+L+3)}{4(L+3)(2L+5)}\alpha_2^2(L+2)(L+1)(2N-L),$$

where L is the angular momentum and N is the boson number, which is equal to half the number of valence nucleons (proton and neutrons). From the given experimental value $B(\text{E2})$ of transition ($2^+ \to 0^+$), one can calculate the value of the parameter α_2^2 for each isotope, where α_2^2 indicates square of effective charge. This value is used to calculate the reduced transition probabilities $B(\text{E2}; L+2 \to L)\downarrow$. Table 3 indicates reduced transition probabilities for all nuclei and the comparisons of calculation values

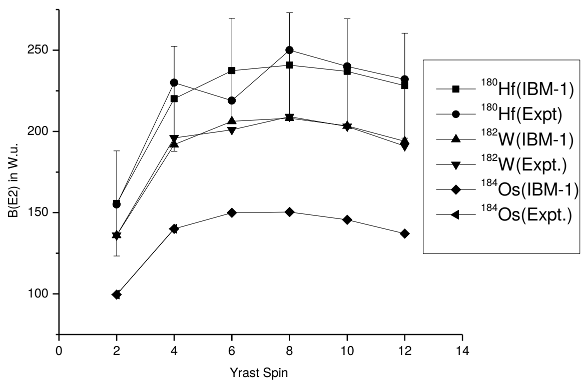

Fig. 3 B(E2) values in W.u. as a function of yrast transition spin for Hf, W, and Os isotopes with neutron $N = 108$.

with the experimental data are excellent. Fig. 3 shows B(E2) values in Weisskopf units (W.u.) as a function of yrast transition spin for Hf, W, and Os isotopes with neutron $N = 108$. It is shown that B(E2) values are maximum for the transition (8^+ to 6^+) in each nucleus. Moreover, the reduced transition probabilities are decrease as proton number increases towards the shell $Z = 82$ and there are good agreement between the IBM-1 and the experimental data.

Quadrupole moments and deformation parameter

The calculation of quadrupole moments is very important to understand the deformation about prolate or oblate shape. The quadrupole moments (Q_0) and $Q_{2_1^+}$ of nuclei can be calculated[3,23] by

$$Q_0 = \alpha_2 \left(\frac{16\pi}{40}\right)^{1/2} (4N+3),$$

$$Q_{2_1^+} = \alpha_2 \left(\frac{16\pi}{40}\right)^{1/2} \tfrac{2}{7}(4N+3).$$

The quadrupole deformation parameters β are calculated[24] by

$$\beta = [B(\mathrm{E2})\uparrow]^{1/2}[3ZeR_0^2/4\pi]^{-1},$$

where Z is the atomic number, R_0 is the average radius of nucleus

$$R_0^2 = 0.0144A^{2/3}b.$$

Table 4 shows quadrupole moment and deformation parameter of even Hf, W, and Os. The calculation of intrinsic quadrupole moments, and deformation parameter are in good agreement to the previous results[20–22,24]. Fig. 4 shows that the Q_0 and $Q_{2_1^+}$ values, as a function of atomic number, are consistent with the experimental data. The intrinsic quadrupole moments are rapidly decreasing from atomic number 72–76.

Table 4 Quadrupole moment and deformation parameter of even ^{180}Hf, ^{182}W, and ^{184}Os nuclei.

Nucl.	β_{exp}	β_{cal}	$Q_{0,exp}$	$Q_{0,cal}$	$Q_{2^+,exp}$	$Q_{2^+,cal}$
^{180}Hf	0.27(35)	0.27	6.9(9)	6.86	−2.0(2)	−1.96
^{182}W	0.25(24)	0.25	6.5(6)	6.45	−2.1(35)	−1.84
^{184}Os	0.21(5)	0.21	5.7(14)	5.58	−2.4(11)	−1.59

β_{exp}, $Q_{0,exp}$: Ref. 25; $Q_{2^+,exp}$: Refs. 20–22.

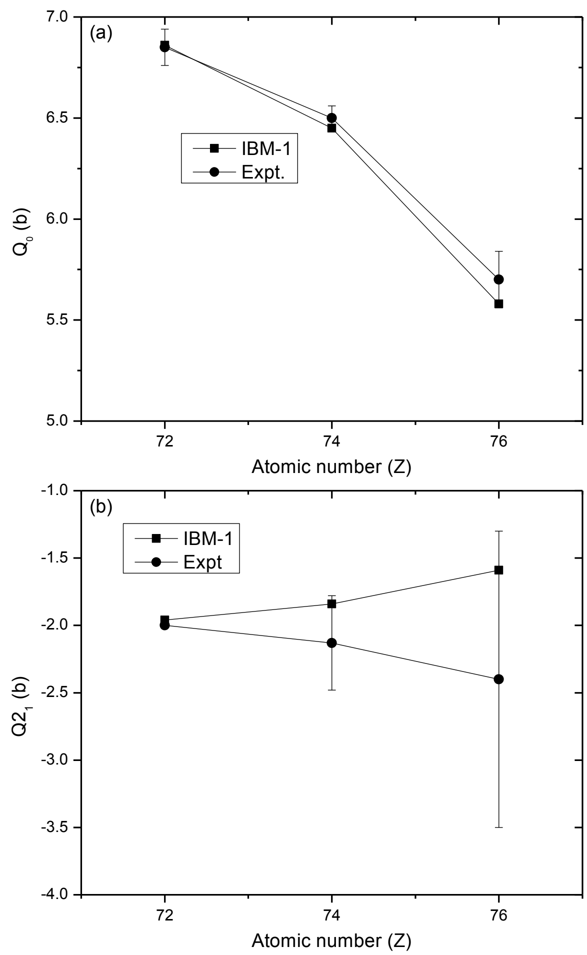

Fig. 4 (a) Q_0 and (b) $Q2_1$ as a function of atomic number (Z) in ^{180}Hf, ^{182}W, and ^{184}Os nuclei.

Back-bending

The positive parity yrast levels are connected by a sequence of stretched E2 transition with energies which increase smoothly except around the back-bends. The transition energy $\Delta E_{I,I-2}$ should increase linearly with I for the constant rotor as $\Delta E_{I,I-2} = I/2\vartheta(4I-2)$ does not increase, but decreases for certain I values.

The relation between the moment of inertia (ϑ) and gamma energy E_γ is given by

$$2\vartheta/\hbar^2 = \frac{2(2I-1)}{E(I)-E(I-2)} = \frac{4I-2}{E_\gamma}, \qquad (3)$$

and the relation between E_γ and $\hbar\omega$ is given by

$$\hbar\omega = \frac{E(I)-E(I-2)}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}}, = \frac{E_\gamma}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}}. \qquad (4)$$

The moment of inertia $2\vartheta/\hbar^2$ and rotational frequency $\hbar\omega$ have been calculated from (3) and (4), respectively. Excitation energy, moment of inertia, and square of rotational frequency for even ^{180}Hf, ^{182}W, and ^{184}Os nuclei are presented in Table 5. The ground state bands up to 12 units of angular momentum are investigated for moment of inertia in Hf, W, and Os with $N = 108$. The moments of inertia are plotted versus square of rotational energy in Fig. 5. Usually, in the lowest order according to the variable moment of inertia (VMI) model[25] this should give a straight line in the plot of inertia as a function of ω^2. It is seen that ^{182}W and ^{180}Hf nuclei show back bending at $I = 4^+$ and $I = 2^+$, respectively. But there is no back bending for ^{184}Os nucleus. The results are presented on collective $\Delta I = 2$ ground band level sequence for the variation of shapes for $Z = 76$, 74, and 72 with even neutron $N = 108$. The back-bending phenomena appear clearly in the diagram $2\vartheta/\hbar^2$ versus $(\hbar\omega)^2$. The back bending phenomenon can be phenomenological reproduced as an effect due to the crossing of two bands.

Table 5 Excitation energy (E_γ), moment of inertia ($2\vartheta/\hbar^2$), and square of rotational frequency for $(\hbar\omega)^2$ even ^{180}Hf, ^{182}W, and ^{184}Os nuclei.

Nucl.	I	$I(I+1)$	Transition level	E_γ (MeV)	$2\vartheta/\hbar^2$ (MeV^{-1})	$(\hbar\omega)^2$ (MeV^2)
^{180}Hf	2	6	$2^+ \to 0^+$	0.0933	42.877	0.1087
	4	20	$4^+ \to 2^+$	0.2153	65.026	0.0464
	6	42	$6^+ \to 4^+$	0.3323	66.205	0.1104
	8	72	$8^+ \to 6^+$	0.443	67.720	0.1962
	10	110	$10^+ \to 8^+$	0.5465	69.533	0.2987
	12	156	$12^+ \to 10^+$	0.6684	68.821	0.4468
	14	210	$14^+ \to 12^+$	0.5359	100.755	0.2872
^{182}W	2	6	$2^+ \to 0^+$	0.1001	59.940	0.0100
	4	20	$4^+ \to 2^+$	0.6606	21.193	0.4364
	6	42	$6^+ \to 4^+$	0.3509	62.696	0.1231
	8	72	$8^+ \to 6^+$	0.4639	64.669	0.2152
	10	110	$10^+ \to 8^+$	0.5676	66.949	0.3222
	12	156	$12^+ \to 10^+$	0.6606	69.634	0.4364
^{184}Os	2	6	$2^+ \to 0^+$	0.1197	50.125	0.0143
	4	20	$4^+ \to 2^+$	0.2639	53.050	0.0696
	6	42	$6^+ \to 4^+$	0.3904	56.352	0.1524
	8	72	$8^+ \to 6^+$	0.5006	59.928	0.3557
	10	110	$10^+ \to 8^+$	0.5964	63.716	0.3557
	12	156	$12^+ \to 10^+$	0.6764	68.007	0.4575

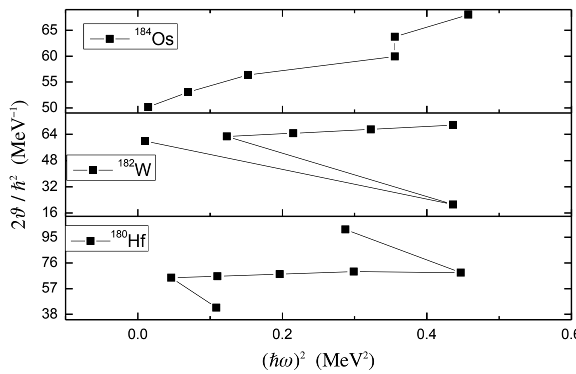

Fig. 5 Plot of the inertia $2\vartheta/\hbar^2$ as a function of $(\hbar\omega)^2$ in ^{180}Hf, ^{182}W, and ^{184}Os nuclei.

Conclusions

We report evolution of positive parity yrast levels, reduced transition B(E2) and quadrupole moments of even-even ^{180}Hf, ^{182}W, and ^{184}Os nuclei by IBM-1 and compared with previous experimental values. The predicted low-lying levels, the reduced probabilities and quadrupole moments were consistent with the experimental results. The back-bending phenomena of those nuclei appear clearly in the plot of $2\vartheta/\hbar^2$ versus $(\hbar\omega)^2$. The analytic IBM-1 calculation of those values of even-even Hf, W, and Os nuclei with $N = 108$ have been performed in the SU(3) deformation character. The results are extremely useful for compiling nuclear data table.

Acknowledgements: This work was supported by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant no. (662-853-D1435). The authors therefore, gratefully acknowledge the DSR technical and financial support.

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