ScienceAsia 28 (2002) : 393-400

Phase Space Study of the Synchrotron Oscillation and Radiation Damping of the Longitudinal and Transverse Oscillations

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Received 17 May 2001
Accepted 1 Apr 2002

ABSTRACT Important phase space parameters of the synchrotron oscillation and damping times for the synchrotron and betatron oscillations have been calculated for the storage ring of the Siam Photon Source. The work is implemented by the use of the computer programs in Visual Basic. The validity of the calculation has been confirmed by the comparison of the numerical data with the corresponding values obtained by the analytical calculation. The results obtained present the important basic data of the Siam Photon Light Source. The evaluation procedure and the detailed analysis of the results are described.

KEYWORDS: synchrotron oscillation, betatron oscillation, radio frequency (RF) acceleration, momentum acceptance, radiation damping.

INTRODUCTION

The Siam Photon Laboratory owns an accelerator complex named the Siam Photon Source consisting of a 40-MeV linac, a 1-GeV booster synchrotron and a 1-GeV storage ring. Some of the important parameters of the Siam Photon ring are given in Table 1.

An electron makes circular motion in a bending magnetic field around the storage ring. If the kinetic energy of the electron is E_0 and the magnetic field strength is B_0, the radius of the circle ρ_0, is determined by the balance between the Lorentz force and the centrifugal force. ρ_0 is given as

$$\rho_0 = E_0 / e\, c\, B_0 \qquad (1)$$

Table 1. Main parameters of the storage ring in the Siam Photon Source.

Electron energy, E_0	1.0 GeV
Circumference, C	81.3 m
Bending radius, ρ	2.78 m
Momentum compaction factor, α	0.0214
Betatron wave numbers; ν_x, ν_y	4.758, 2.823
RF frequency, f_{rf}	118 MHz
RF voltage, V	100 kV
Harmonic number, h	32

where, e is the electron charge, c the speed of light in vacuum, and ρ_0 the radius of curvature of the electron trajectory in the field of the bending magnet. In a storage ring, the bending orbit defined by Eq (1) connects with a linear trajectory in an adjacent drift space called the straight section. The trajectory formed in this way is called the ideal orbit. Electrons with different energies move along the different trajectories that are closed, if the orbit oscillation to be mentioned later is not taken into account. The trajectories are called the closed orbits. Electrons moving along the bent orbits with very high speeds (almost equal to the light velocity) emit synchrotron radiation towards the direction of electron motion and lose their energies. Consequently they are retarded along the electron orbit. The storage ring is equipped with a RF cavity to accelerate the electrons in the longitudinal direction and to compensate the lost energies of electrons in motion. Since the acceleration is made longitudinally electrons make longitudinal oscillation along the closed orbits. The longitudinal oscillation can be understood from a different viewpoint. In a storage ring, an electron with a higher energy moves along the longer closed orbit. If the electron energy oscillates, the electron makes an oscillatory motion along the electron orbit. This longitudinal oscillation is referred to as the synchrotron oscillation. The quadrupole magnets are installed in a storage ring to focus the divergent electron beam. They give restoring force to electrons that deviate from the ideal orbit. Because of the restoring force electrons move along the trajectories oscillating transversely around the ideal orbit. This transverse oscillation is referred to as the betatron oscillation.

Let the RF field vary as $V_{rf} = V_0 \sin \omega t$ and the synchronous electron arrive at the RF cavity at $\omega t = \Phi_s$. If an electron with momentum p_0 and revolution time T_0 makes the revolution synchronized with RF phase $\Phi = \Phi_s$, it is called as a synchronous electron. Synchronous phase Φ_s is defined as a phase at which an electron gains the energy exactly equal to average energy loss per turn U_0 (energy loss at E_0). This leads to no net change of energy per turn. This is shown in Fig 1 that the synchronous electron is neither accelerated nor decelerated virtually during one revolution. An electron with momentum p slightly different from p_0 is on a closed orbit different from the ideal orbit. Such an orbit is referred to as the off-momentum closed orbit and the fractional momentum deviation d is expressed as $(p-p_0)/p_0$.

The synchronous RF phase Φ_s must be selected properly to obtain stable synchrotron oscillations and it should be in the region $\pi/2 < \Phi_s < \pi$ for $\alpha > 0$. α is the momentum compaction factor and the value is given in Table 1. Φ_s is known as the stable fixed point and represents the motion of synchronous electron. The electrons arriving at the RF cavity around this phase angle can make stable synchrotron oscillations. $(\pi - \Phi_s)$ is known as unstable fixed point (UFP) and the electrons arriving at the RF cavity at this phase angle make the unstable motion. The torus that passes through UFP is called separatrix, which separates the longitudinal phase space into stable and unstable regions. The stable area inside the separatrix is known as the RF Bucket area. The electrons inside the bucket execute stable phase oscillations, and gain energy along with the synchronous electron. The electrons outside the bucket will slip into the wrong phase relative to RF wave and will not be accelerated. There will be many RF buckets in a typical storage ring and one such RF bucket is shown in Fig 1. The synchrotron oscillation in phase space with reference to the energy gain function $eV(t)$ illustrating the Φ_s, $(\pi - \Phi_s)$ and phase stable region is shown in Fig 1.

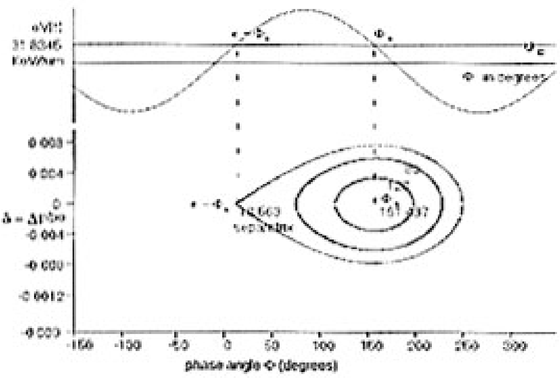

Fig 1. Synchrotron oscillation in the phase space in relation to the energy gain function $eV(t)$. The curves illustrated the synchronous phase angle and the stable region of the synchrotron oscillation. Phase space trajectories obtained at different phase angles with $\delta = 0$ is shown here.

In a storage ring, electrons pass through the RF accelerating field periodically at every T_0 seconds and perform synchrotron oscillations with frequency f_s about the synchronous phase. In the course of performing synchrotron oscillations, electrons on the trajectories reach maximum deviations in the momentum and the phase. The maximum momentum deviation and the maximum phase deviation define the stability limits of the RF bucket.[1, 2]

The damping of the synchrotron oscillation is caused by the fact that the synchrotron radiation power or the rate of change of energy loss is dependent on the electron energy. The total energy radiated in one revolution can be written as follows.

$$U_{rad} = C\gamma E^2 B^2/\rho \;^{a}\; C\gamma E^4/\rho \qquad (2)$$

where $C\gamma$ is the radiation constant, E is the energy of the electron and ρ is the bending radius. This shows the energy loss is proportional to fourth power of the electron energy. So the higher energy electrons lose (radiate) more energy than the lower energy electrons. The synchrotron oscillation is damped at a rate proportional to dU_{rad}/dE. The average energy loss is compensated by the longitudinal electric field of the RF cavity. The damped synchrotron oscillation is expressed in terms of angular synchrotron frequency ω_s and damping coefficient α_ε as given below.

$$\tau(t) = \tau_0\, e^{-(\alpha_\varepsilon - i\omega_s)t} \quad \text{and} \quad \varepsilon(t) = \varepsilon_0\, e^{-(\alpha_\varepsilon - i\omega_s)t} \qquad (3)$$

where τ and ε are time displacement and energy deviation with reference to the synchronous particle. τ_0 and ε_0 are complex constants determined by the initial conditions.

The damping of betatron oscillation arises from the combination of energy loss due to synchrotron radiation in the direction of particle motion (energy loss is correlated to the loss in transverse momentum) and the energy gain through RF accelerating field in the longitudinal direction. The vertical betatron oscillation is also damped. When an electron loses an amount of energy U_{rad} by radiation, the momentum vector p changes by Δp such that the change is parallel and opposite to the direction of momentum vector p. During this process, betatron amplitude is unchanged. When the energy loss is compensated by the RF accelerating field in the longitudinal direction, only the longitudinal component of momentum is affected. The RF acceleration does not change the position y, but the slope y' is decreased by the increment of the longitudinal momentum ie,

$$y' \rightarrow (1 - \Delta p/p_0)\, y' \qquad (4)$$

Thus, there is a corresponding change in the amplitude of betatron oscillation. Therefore, energy loss alone does not result in the phase space damping.

In case of the horizontal betatron oscillation, we have to consider the closed orbit changes at an instant when a photon, which is emitted as the non-vanishing dispersion function η_x in dipole magnets, causes this change. Since we have assumed that the electrons are moving in the horizontal plane, no bending occurs in the vertical direction. Thus η_y is zero. This is the reason for ignoring the dispersion effect in the case of the vertical betatron motion. The electron displacement from the ideal orbit is given by $x = x_\varepsilon + x_\beta$ where x_β is the betatron displacement and x_ε is displacement of off-energy closed orbit. When the energy of the electron drops by ΔE owing to the emission of synchrotron radiation, there is a change in x_ε by an amount $\Delta x_\varepsilon = \eta \Delta E/E_0$. Since the position of the electron is not changed by finite change in momentum, the total x does not change, but there is a compensatory change in x_β (increase of betatron displacement). So the average energy loss due to synchrotron radiation (on the average) gives rise to some anti-damping, ie, growth in the horizontal betatron oscillation amplitude. The effect of the RF acceleration and its contribution to the damping of the betatron oscillations is exactly the same as that of the vertical oscillations as explained earlier. The total effect in one revolution can be obtained by adding the contributions from the radiation loss and the RF acceleration.

The purpose of this work is to study the motion of an electron making the synchrotron oscillation and the betatron oscillation with damping. The practical calculation is made in the longitudinal and transverse phase space by developing the Visual Basic computer programs. Accurate results are obtained in calculating the different parameters as mentioned below. This led us to a better understanding of the results. In addition to this, the dependence of the maximum momentum deviation and the maximum phase deviation on the synchronous phase angle as well as the variation of synchrotron oscillation frequency with the maximum momentum deviation are investigated.

The following parameters have been calculated for the 1-GeV storage ring of the Siam Photon Source.

1. Synchrotron oscillation frequency (f_s)
2. Maximum momentum deviation (δ_{max})
3. Maximum phase deviation
4. Damping time for the synchrotron oscillation (τ_ε)
5. Damping time for the vertical betatron oscillation (τ_y)
6. Damping time for the horizontal betatron oscillation (τ_x)

Computational Method

Longitudinal equations of motion are used to calculate the parameters like the synchrotron oscillation frequency, the maximum momentum deviation and the phase deviation. The synchrotron oscillation in phase space is more realistically described by the following mapping equations[3], which are derived from the synchrotron equations of motion.

$$\delta_{n+1} = \delta_n + (eV/\beta^2\, E_0)(\sin\Phi_n - \sin\Phi_s) \qquad (5)$$

$$\Phi_{n+1} = \Phi_n + 2\pi\, h\alpha\, \delta_{n+1} \qquad (6)$$

where δ_n is the deviation of the electron momentum from that of the synchronous electron at n_{th} revolution, e the charge of electron, V the peak RF voltage, β the velocity of the electron relative to speed of light, E_0 the energy of synchronous electron, Φ_n the phase angle of the RF wave seen by the off-momentum electron at n_{th} revolution, Φ_s the phase angle of the RF wave seen by the synchronous electron, h the harmonic number and α the momentum compaction factor.

The energy lost by the synchronous electron, U_0, is found by using Eq (2) with $E = E_0$. The energy gained by the synchronous electron is $eV\sin\Phi_s$, where Φ_s is determined by equating $U_0 = eV\sin\Phi_s$ and $\Phi_s = 161.437^0$ in the present case.

An accelerating RF bucket with $\Phi_s = 161.437^0$ is shown in Fig 1. The phase space trajectories around this point are ellipses. The motion in the neighborhood of this point has small amplitude. The point corresponding to the set of initial values (Φ_0, δ_0) as (161.437, 0) is known as the stable fixed point (SFP), and represents the motion of the synchronous electron. Fig 1 shows the different phase space trajectories obtained for the initial values (Φ_0, δ_0) corresponding to (18.563, 0), (80, 0), (120, 0) and (161.437, 0).

(18.563, 0) is the unstable fixed point (UFP) and the phase space trajectories near UFP depart from elliptical shapes and the electron motion is less linear. This is a large amplitude motion.

The synchrotron oscillation frequency, an important characteristic parameter of the synchrotron oscillation, is determined by the number of revolutions, *n*, taken by the electron to complete one period of synchrotron oscillation. If the time taken by the electron to complete one revolution is T_0, $1/(nT_0)$ gives the frequency of synchrotron oscillation.

The maximum momentum deviation and the maximum phase deviation are characteristic parameters of the separatrix, and the stability limits of the RF bucket are defined by these parameters. The phase space trajectory of the electron, (Φ, δ), depends on the point (Φ_0, δ_0) at which it starts. The maximum momentum deviation of the electron, δ_{max}, is obtained by comparing the maximum values of δ for different trajectories near the separatrix (large amplitude oscillations occur near separatrix). The maximum phase deviation is calculated in the same way. This is also known as bucket length.

The energy radiated or the energy loss depends on the energy of the electron that changes for each revolution, and the energy gained through the RF acceleration depends on the phase angle of the RF wave seen by the electron. Using the longitudinal mapping equations (refer to Eqs (5) and (6)) with the calculation principle described above, the radiation damping in longitudinal phase space has been studied. The following assumptions are made in damping time calculations:

1. As U_0 is typically smaller than the electron energy by a factor 10^3 or more, we consider only the effects that occur over many revolutions by neglecting the changes in electron energy during one revolution.
2. The energy, U_{rad}, radiated in one revolution can be obtained by integrating the instantaneous power radiated by a relativistic electron with respect to time. The instantaneous power is proportional to the square of the energy and the magnetic field strength, so U_{rad} is proportional to E^2B^2.

The damping time te is the time during which the electron energy is reduced to *1/e* of its maximum initial energy. The number of revolutions, *n*, taken by the electron to damp down its oscillation energy to *1/e* of its maximum is found by the program and the damping time is calculated as given below.

$$A(t) = A_m\, e^{-t/\tau}{}_{\varepsilon} = A_m\, e^{-t/nT}{}_0 \qquad (7)$$

Where T_0 is the revolution period, $A(t)$ the amplitude as a function of time, *and* A_m the maximum amplitude. For the betatron oscillation, the phase space coordinates at n_{th} revolution are obtained by multiplying the transfer matrix for one complete revolution with the phase space coordinates at $(n\text{-}1)_{th}$ revolution. Transfer matrix $M(s + C/s)$ corresponding to one revolution, with *s* as azimuthal coordinate and *C* as circumference, can be expressed as follows.

$$\begin{vmatrix} \cos 2\pi\nu_y + \alpha_y \sin 2\pi\nu_y & \beta \sin 2\pi\nu_y \\ -\gamma \sin 2\pi\nu_y & \cos 2\pi\nu_y - \alpha_y \sin 2\pi\nu_y \end{vmatrix} \qquad (8)$$

The phase space ellipse that represents the betatron oscillation is obtained by plotting the variation of *y* or *x* (displacement) and *y'* or *x'* (slope) for *n* revolutions using the transfer matrix corresponding to one revolution. Damping times for the betatron oscillation are calculated by considering the damping of longitudinal oscillation and also by using the Eq (4). Computer programs developed in Visual Basic are used for the calculation of different parameters.

Results

Synchrotron oscillation frequency

Let us consider the small amplitude oscillation, in which the phase does not deviate much from Φ_s and the fractional momentum deviation, δ, is very small. The small amplitude oscillation frequency is calculated by finding the number of revolutions in one period of synchrotron oscillation. The value obtained by the program is 11.827 kHz. The small amplitude oscillation frequency is given analytically[3] in Eq (9).

$$f_s = f_0\,(\, h\ e\ V\,|\_\ cos\ \_{}_s|\,/\,2\ p\ b^2E_0\,)^{1/2} \quad (9)$$

Using the parameters of the Siam Photon ring (f_o = 3.6875 MHz, β = 1, Φ_s = 161.437^0) in Eq (9), the synchrotron oscillation frequency is obtained as 11.84 kHz. In case of the large amplitude oscillation, the electron takes higher values of δ and moves through longer path. So the number of revolutions, n, taken by the electron to complete one period of synchrotron oscillation is greater than that of the small amplitudes. This is shown in Fig 2 by plotting the variation of synchrotron oscillation frequency, f_s, with the amplitude or height of the RF bucket, δ_{max}. The data exhibits two characteristic features:

1. The synchrotron oscillation frequency of the small amplitude oscillation is higher than that of the large amplitude oscillation.
2. A steep edge occurs near $\delta_{max} = 0.007$

The low amplitude oscillation takes place when the RF acceleration is made only by the linear part of the RF voltage. The large amplitude oscillation occurs when the acceleration is made even by the non-linear part of the RF voltage. The plot in Fig 2 shows that the synchrotron oscillation frequency becomes very low when the maximum momentum deviation reaches its higher limit. The region of maximum momentum deviation, in which the synchrotron oscillation frequency drops rapidly with δ_{max}, is considered the unstable oscillation region.

Maximum momentum deviation

Fig 3 shows the separatrix plotted as the momentum deviation δ versus the phase angle of the RF field. The separatrix illustrated in the phase space of ε as energy deviation from the synchronous electron versus τ as difference of arrival time from that of synchronous electron has a shape similar to the curve shown in Fig 3. The maximum momentum deviation δ_{max} has been calculated according to the method given in Sec 2. The obtained value is 7.157E-3 for the initial values of (Φ_0, δ_0) as (18.563, 0). The δ_{max} that occurs near the separatrix is also known as the bucket height. The above results of the computer simulation can be compared with the values obtained by the analytical expressions[3] in Eqs (10) and (11). Using the parameters of the Siam Photon storage ring and $\Phi_s = 161.437^0$, the obtained value for momentum acceptance δ_{max} is 7.157E-3, it is observed that

$$\delta_{max} = (2eV/\pi\, \beta^2\, \alpha\, E\, h)^{1/2}\ y(\Phi_s) \qquad (10)$$

$$\text{where, } y(\Phi_s) = \left| \cos\Phi_s - (\pi - 2\Phi_s)/2 \sin\Phi_s \right|^{1/2} \qquad (11)$$

Momentum acceptance is a function of synchronous phase angle, Φ_s. The maximum momentum deviation or the momentum acceptance increases as the synchronous phase angle Φ_s increases. The relationship between the maximum momentum deviation and the synchronous phase angle is shown in Fig 4.

Maximum phase deviation

The maximum phase deviation has been calculated by the computer simulation and the result is 231.624^0 or 4.0426 radians for the initial values of (Φ_0, δ_0) as (18.563^0, 0). The result is also shown in Fig 3. This parameter shows the boundary of the RF bucket in terms of Φ, ie, the maximum phase seen by the off-momentum electron with reference to that of the longitudinal electric field. It is compared with the analytical calculation using the expressions[3] given in Eqs (12) and (13). The maximum phase deviation is expressed as

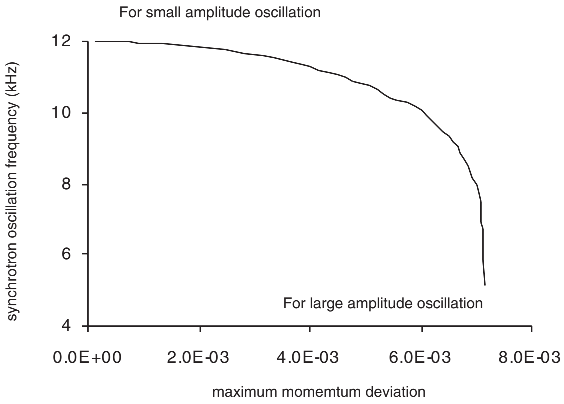

Fig 2. The variation of synchrotron oscillation frequency with the maximum momentum deviation δ_{max}.

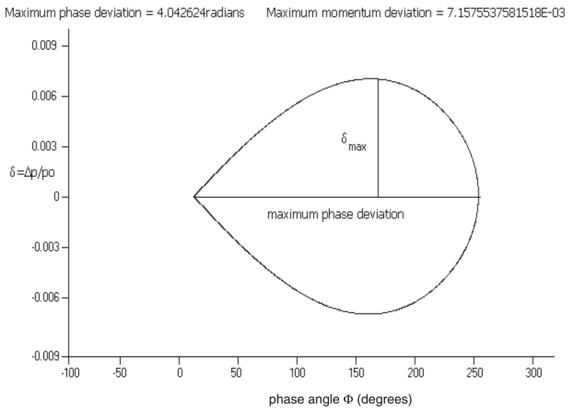

Fig 3. The phase space diagram of the synchrotron oscillation indicating the maximum momentum deviation and the maximum phase deviation.

$$|(\pi-\Phi_s)-\Phi_u| \tag{12}$$

where Φ_u can be obtained by solving the equation

$$\cos\Phi_u+\Phi_u\sin\Phi_s=\cos\Phi_s+(\pi-\Phi_s)\sin\Phi_s \tag{13}$$

For Φ_s = 161.437^0 or 2.8176 radians, Φ_u equals to 250.186^0 or 4.366 radians and the maximum phase deviation is 231.623^0 or 4.04259 radians. If an electron is injected into a storage ring with the momentum and phase deviations below the limits given by their maximum values, it circulates on a curved trajectory within the bunch. If it is injected outside the separatrix (outside the RF bucket) it is lost. The relationship between the maximum phase deviation and the synchronous phase angle is shown in Fig 5. It is observed in Fig 5 that the maximum phase deviation increases with the synchronous phase angle.

Radiation damping of the synchrotron oscillation

The radiation damping time is calculated and the obtained value is 8.19 ms. The result is also shown in Fig 6. Since $dU_{rad}/dE > 0$, the electron loses energy in the upper part of its path in phase space relative to the synchronous electron while it gains energy in the lower part of its path. The analysis shows that the synchrotron oscillation keeps damping continuously. Thus with damping, the size of the ellipse decreases and phase space trajectory is an inward spiral shown in Fig 6. The center of the spiral motion represents the synchronous electron or the center of the bucket. In practice, the emission of photons can excite the oscillation. Thus equilibrium is attained in a certain time. The result is compared with the analytical calculation using the expressions[4] given in Eqs (14) and (15).

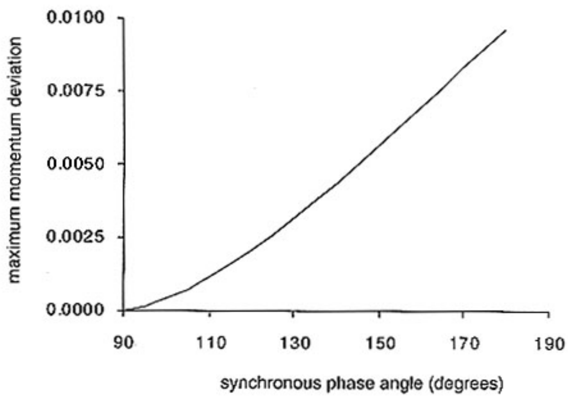

Fig 4. The variation of the maximum momentum deviation, δ_{max}, with the synchronous phase angle.

$$\alpha_\varepsilon=(Uo/2\,Eo\,To)(2+D);\tau_\varepsilon=1/\alpha_\varepsilon \tag{14}$$

$$\text{where, } D=R\;\alpha/\rho \tag{15}$$

Here, α_ε is the damping coefficient. In Eqs (14) and (15), (2 + D) denotes the damping partition number in case of isomagnetic separated function lattice, R is the effective radius of the ring ie, $C/2\pi$. By considering the parameters of the Siam Photon storage ring, the obtained value of U_0 is 31.8345 keV/turn with $C\gamma$ = 8.85E-5 m/(GeV)^3. With these parameters the damping coefficient α_ε is obtained as 123.45 s^{-1} with T_0 = 0.271 μs and the damping time as τ_ε = 8.1msec.

Damping of the vertical betatron oscillation

The damped betatron motion is obtained by using the principle explained in Sec 1. The damping is caused by the RF acceleration. Through the coupling with the longitudinal mode, the radiation damping of the betatron oscillation occurs. With this concept,

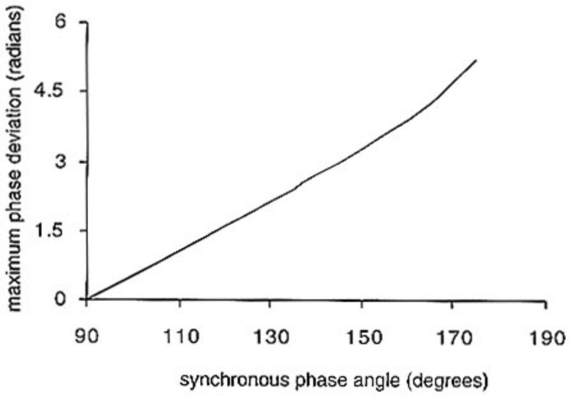

Fig 5. The variation of the maximum phase deviation with the synchronous phase angle Φ_s.

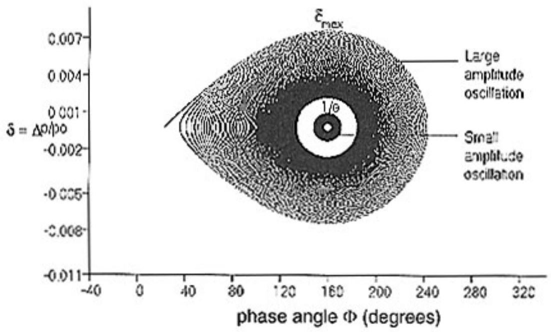

Fig 6. Radiation damping of the synchrotron oscillation for the small and large amplitude oscillation.

a program has been developed to study the radiation damping of the vertical betatron oscillation in the phase space. The damped betatron oscillation in the phase space makes the inward spiral as shown in Fig 7. In the present case, damping time calculation is done for the ideal situation by neglecting the magnet imperfections. Another important parameter is that the oscillation amplitude corresponds to the Courant-Snyder integral.

The damping time is taken as the time at which the amplitude of the oscillation is reduced to 1/e from its maximum value. It has been found that the damping time varies with the starting position of the electron motion ie, y_0 and y_0'. The calculated damping time is 17.02 msec(maximum). The value is confirmed to be correct by comparing the value obtained with the analytical expression[5] given in Eq (16).

$$\alpha_y = Uo/2\,Eo\,To;\ \tau_y = 1/\alpha_y \qquad (16)$$

Using the parameters of the Siam Photon Source with U_0 equals to 31.8345 keV/turn, T_0 = 0.271 μs, the damping coefficient is obtained as α_y =58.7353 s^{-1} and the damping time as τ_y = 17.02 msec.

Damping of the horizontal betatron oscillation

The damping time for the horizontal betatron motion has been calculated in the same way as explained in the case of vertical betatron oscillation. Fig 8 shows the damping of horizontal betatron oscillation and the damping time is found as 17.038 msec(maximum). Damping time is taken as the time at which the amplitude of the oscillation is reduced to 1/e from its maximum value. It has been observed that variation of damping time is very much dependent on the initial values of the particle position ie, x_0 and x_0'. In this calculation dispersion is set to zero which is not true in case of horizontal betatron oscillation. The variation in the energy loss during one cycle is assumed as constant in this calculation. Considering the damping due to the RF acceleration and anti-damping due to non-zero dispersion function caused by synchrotron radiation, the damping coefficient[6] is given as

$$\alpha_x = (1-D)\,Uo/2\,Eo\,To;\ \tau_x = 1/\alpha_x \qquad (17)$$

where D is given by Eq (15). Using the parameters of Siam Photon Source, the damping coefficient is obtained as α_x = 52.8846s^{-1}(with D = 0.09961) and τ_x = 18.9msec.

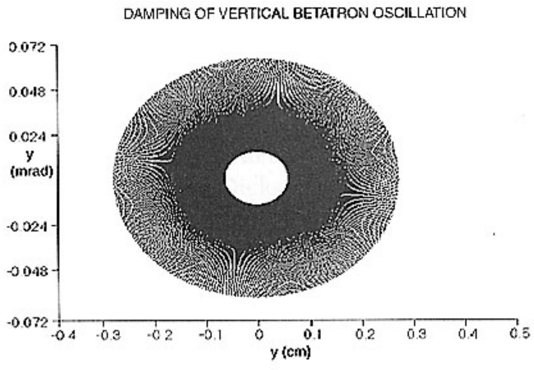

Fig 7. Damping of the vertical betatron oscillation.

Summary and discussion

Based on the calculations performed in the present work, some parameters of the Siam Photon Storage Ring are evaluated and the obtained values are listed in Table 2.

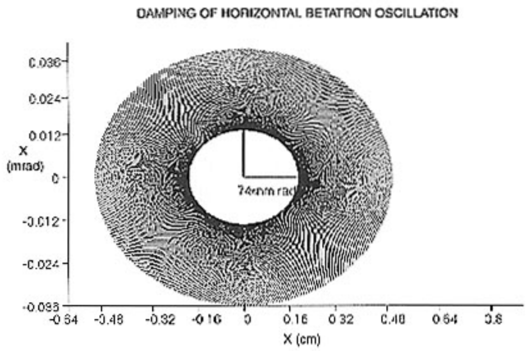

Fig 8. Damping of the horizontal betatron oscillation.

Table 2. Results of the different parameters obtained by the computer simulation.

Small amplitude synchrotron oscillation frequency	11.83 KHz
Momentum acceptance of the RF system	0.007158
Maximum phase deviation radians	4.0426
Damping time for synchrotron oscillation	8.19 msec
Damping time for vertical betatron oscillation	17.05 msec
Damping time for Horizontal betatron oscillation	17.038 msec

The results are in agreement with the values calculated analytically. It has been observed that the synchrotron oscillation frequency varies inversely with the δ_{max} (maximum momentum deviation), and the maximum phase deviation increases linearly with the synchronous phase angle. These are in agreement with the theory. The damping time for the betatron oscillation is calculated with the assumption that energy loss during one revolution is constant. In case of the horizontal betatron oscillation the effect of dispersion is not taken into account. These are the factors that could have caused error in the calculation of the damping time. For the horizontal betatron oscillation, the area of the phase space ellipse after damping equals to the natural emittance.

In a storage ring, the emission of photons occurs in a stochastic way. This leads to the fluctuation of the photon emision and then the energy deviation, which causes the excitation of the oscillation. In the case of vertical betatron oscillation, the excitation of the oscillation by the emission of photons is brought about only by the second order effect which is the momentum recoil to electrons by the photon momentum. Thus the resulting oscillation energy spread is very small as compared with the case of the horizontal oscillation. Practically the energy spread of the vertical oscillation occurs due to the coupling between the vertical and horizontal betatron oscillations. All these practical situations have been ignored in the present work. However, the data obtained here will be used in the calculation of practical parameters. The phase space ellipse and the separatrix obtained here are also important basic data. They are used in finding the appropriate RF acceleration bucket, that is necessary for the commissioning of the machine.

References

1. Cole FT (1985) Longitudinal motion in circular accelerators. 5th annual US. Particle Accelerator school, Stanford Linear Accelerator center.
2. Wiedemann H (1999) Particle Accelerator Physics 1. In: *Basic principles and Linear beam dynamics*, *2nd ed.* Springer-verlag.
3. Lee SY (1999) Accelerator Physics. *2nd ed.* World Scientific Publishing Company Pvt Limited.
4. Krinsky S, Perlman MN and Watson RE (1983) Handbook on synchrotron radiation: Characteristics of synchrotron radiation and its sources. Vol 1a.
5. Sands M (1970) The physics of electron storage rings. *SLAC-121*.
6. Hubner K (1986) Radiation damping. *European organisation for nuclear research (CERN accelerator school).*