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Modeling of Aggregate Stiffness and Its Effect on Shrinkage of Concrete

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ABSTRACT This paper is aimed to propose a model for simulating aggregate phase stiffness and its effect on concrete shrinkage based on the concept of concrete as a two-phase material (paste phase and aggregate phase). Regarding the effect of aggregate phase, concrete shrinkage is found to be affected by the aggregate content, strain of the aggregate phase and the proportion between coarse aggregates and fine aggregates. The aggregate particles are considered to be uniformly distributed in the concrete and in contact with one another at various contact angles. As a result, the stress of the aggregate phase can be obtained from the summation of stresses of all contact angles. A two-dimensional constitutive model is used for computing the stiffness of the aggregate phase of concrete. The stiffness of the aggregate particle system is then derived from the ratio between stress and strain. Because of concrete shrinkage is restrained by aggregates, the effect of aggregate stiffness on concrete shrinkage is proposed based on the concrete shrinkage model by Tangtermsirikul and Nimityongskul.[1] Verification tests on fine aggregates are conducted on mortar specimens while coarse aggregates and binary mixture of aggregates are conducted on concrete specimens. The verification indicated that the model is effective for deriving the stiffness of aggregate phase and predicting the shrinkage of concrete as well. From the test and analysis results, high aggregate contents induce great stiffness. As a result, it will have effect on small shrinkage.

KEYWORDS: aggregate stiffness, shrinkage, two-phase material, contact angles.

INTRODUCTION

Shrinkage of concrete has a significant influence on the durability and serviceability of the concrete structure. Concrete is composed of paste phase and aggregate phase. However the shrinkage was regarded to occur only in the paste phase whereas the aggregate phase was considered to restrain the shrinkage by their particle interaction. There is much research for predicting the shrinkage taking into account aggregate restraint. One of these is Hobbs' model,[2] which was proposed for estimating drying shrinkage of concrete considering the effect of aggregate. However, this model was not applicable for all ranges of aggregate content and the proportion between fine aggregates and coarse aggregates. The other one is a two-phase material model, taking into account the restraint shrinkage due to aggregate particle interaction, has been proposed by Tangtermsirikul and Nimityongskul.[1] The deformation behavior of aggregate particle system can be predicted using the idea from the two-dimensional constitutive model for solid phase under compression. However, the aggregate stiffness equations used in the model were still macroscopic. So, a more rational microscopic model is proposed in this study. This paper proposes a stiffness model which is applicable to fine aggregates and coarse aggregates, and also covers the stiffness of binary mixture between coarse and fine aggregates.

TWO-PHASE MODEL FOR CONCRETE SHRINKAGE [OVERVIEW]

Concrete is regarded as a two-phase material comprising of paste phase and aggregate phase. Paste phase being the part to undergo shrinkage, consists of all cementitious and powder materials, water, all kinds of mineral and chemical admixtures and air voids. The aggregate phase, considered much more stable in volume, consists of coarse and fine aggregates. Under the assumption that paste phase and aggregate phase develop a full bond, the deformation strain of both phases can be assumed to be equal, and also equal to the deformation strain of concrete. The equilibrium condition between two phases can be written in the internal stress form. Stresses inside concrete are from stress on aggregate phase from paste phase and stress on paste phase from aggregate phase. From the above assumptions based on the properties of concrete, Tangtermsirikul and Nimityongskul[1] proposed an equation for computing shrinkage strain of concrete as

$$\varepsilon_{conc} = \frac{\varepsilon_{po} \cdot E_p \cdot (1-n_a)}{E_p + E_a} \qquad (1)$$

where ε_{conc} is the shrinkage strain of concrete, ε_{po} is the free shrinkage of paste in concrete, E_p is the paste stiffness (kgf/cm^2), E_a is the aggregate stiffness (kgf/cm^2), and n_a is the volume concentration of aggregate which is obtained from

$$n_a = V_a/V_{conc} \qquad (2)$$

where V_a is the volume of aggregate in concrete (m^3) and V_{conc} is the volume of the concrete (m^3).

In this study, the free shrinkage of paste in concrete (ε_{po}) was obtained from the shrinkage test on paste mixtures. A model for predicting free shrinkage of paste has been studied by the authors.[3]

For stiffness of paste (E_p), Yomeyama et al[4] proposed the model which is taken from the effective tensile Young's modulus with no historically sustained tension or compression as

$$E_p = 1.05 \times 10^4 \times (f_c)^{0.474} \qquad (3)$$

where f_c is the compressive strength of paste.

Aggregate Stiffness of Single Material

In simulating the shrinkage of concrete by a two-phase material model, stiffness of aggregate phase has to be obtained as one of the phase properties. The deformational behavior of aggregate particle system can be predicted using the idea from the two-dimensional constitutive model for solid particles under compression. The model which is applicable to fine aggregate and coarse aggregate individually as single materials is firstly proposed based on the contact density concept. Then the concept for deriving stiffness of combined fine and coarse aggregates as binary mixtures was introduced later.

Concept of the Aggregate Stiffness Model for Single Material

Aggregate is considered to be composed of particles which are contacting one another. Each contact has its own contact angle (θ) and the density distribution of the contact angle is assumed to be $\Omega(\theta)$. The $\Omega(\theta)$ can be simply explained as to represent the ratio of the numbers of contact which have angles θ to the total numbers of contact. The force system contains normal force which is caused by the deformation normal to the contact plane and friction force which is due to the deformation parallel to the contact plane. Stress-strain relation that is applied for relating the deformation normal to the contact plane to the corresponding stress is assumed. Friction is treated as dry Coulomb's friction. Contact area increases as the deformation progresses and is affected by particle shape, size and grading of the aggregate. Re-arrangement of particles is also a significant factor especially for low friction particles.

Probability Density Function for Contact Angle

Particles are considered to have a density function for contact angle as in Fig 1. Li and Maekawa[5] proposed a different function for effective contact area to model the shear transfer across crack. Here it is reasonable to assume that there are negligible contact angles which are normal and parallel to the principle strain direction (θ equals 0 and π/2), but most contact angles are nearly or just π/4. Then the function for contact angle is assumed as

$$\Omega(\theta) = \sin 2\theta \qquad (4)$$

$$\int_0^{\pi/2} \Omega(\theta) d\theta = 1.0 \qquad (5)$$

Deformation at a Contact

From the geometry in Fig 2, considering a unit volume, the deformation, ω_θ and δ_θ, can be related to strains (ε_y and ε_z) by coordinate transformation. Then

$$\omega_\theta = \varepsilon_z \cdot \cos\theta + \varepsilon_y \cdot \sin\theta \qquad (6)$$

$$\delta_\theta = \varepsilon_z \cdot \sin\theta - \varepsilon_y \cdot \cos\theta \qquad (7)$$

As the co-ordinate axis coincides with the principal strain directions, the shear strain, ε_{xy}, equals zero.

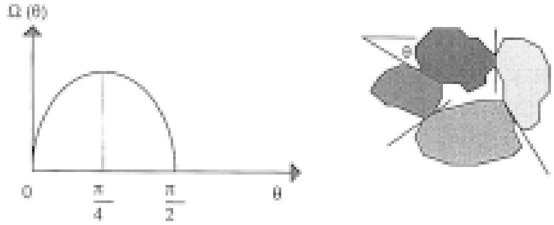

Fig 1. Density Function of Contact Angle.

Constitutive Relation for Normal Direction

In this study, the stress-strain relationship for relating the normal stress ($\sigma_{c\theta}$) to its corresponding deformation (ω_θ) is considered to be linear. The monotonic local stress-strain relation of a contact is assumed to be

$$\sigma_{c\theta} = E_c' \cdot \omega_\theta \tag{8}$$

where E_c' is the stiffness of stress-strain relationship of the contact displacement in the direction normal to the contact plane, and was reasonably assumed to have a value of 2.5 x 10^5 kgf/cm^2 in this study.

Stress in Direction Parallel to Contact Plane

To simplify the problem, the frictional stress (f_θ) is assumed constant independent on slip (δ) as in the following expression

$$f_\theta = \mu. \cdot \sigma_{c\theta} \tag{9}$$

where μ is the coefficient of physical friction between grain of particles. In this study, all of concrete specimens were conducted on water to cement ratio equal to 0.30, μ for crushed limestone coarse aggregate and river sand were assumed to have the values of 0.36 and 0.31, respectively. However, increased amount of water has an effect to lubricate the aggregate particles and reduces the friction along the interfaces of aggregates. As a result, μ is less in the mixes that have a greater water to cement ratio.

Equilibrium Equations

The local force system performing on the contact at angle can be transformed to be the forces ($F_{z\theta}$, $F_{y\theta}$) in the global coordinate system as

$$F_{z\theta} = (\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) \cdot A_{c\theta} \tag{10}$$
$$F_{y\theta} = (\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) \cdot A_{c\theta} \tag{11}$$

where $A_{c\theta}$ is contact area.

Equilibrium is satisfied by integrating the multiplication product of forces with the density function of the contact angle in the global coordinate over contact angles from $\theta = 0$ to $\pi/2$ and equate the integral to the external forces as

$$\sigma_z A_z = \int_0^{\pi/2} \Omega(\theta) \cdot F_{z\theta} \cdot d\theta \tag{12}$$

$$\sigma_y A_y = \int_0^{\pi/2} \Omega(\theta) \cdot F_{y\theta} \cdot d\theta \tag{13}$$

where σ_y and σ_z are the stresses in y and z directions in global coordinate system, respectively; and A_y and A_z are the area normal to y and z directions in global coordinate system, respectively.

Substituting Eq.(10) and Eq.(11) into Eq.(12) and Eq.(13), since $A_y = A_z = 1$, the principal stresses can be obtained as

$$\sigma_z = \int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) \cdot A_{c\theta} \cdot d\theta \tag{14}$$

$$\sigma_y = \int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) \cdot A_{c\theta} \cdot d\theta \tag{15}$$

By introducing the function for contact area ($A_{c\theta}$), Eq.(6), Eq.(7), Eq.(8), Eq.(9), Eq.(14), and Eq.(15) can be solved simultaneously. Subsequently, the two-dimensional stress-strain relationship of the single materials can be obtained as

$$E_{az} = \frac{\sigma_z}{\varepsilon_z} = \frac{\int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) \cdot A_{c\theta} \cdot d\theta}{\varepsilon_z} \tag{16}$$

$$E_{ay} = \frac{\sigma_y}{\varepsilon_y} = \frac{\int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) \cdot A_{c\theta} \cdot d\theta}{\varepsilon_y} \tag{17}$$

where E_{ay} and E_{az} are the aggregate stiffness in y and z directions of the global coordinate, respectively.

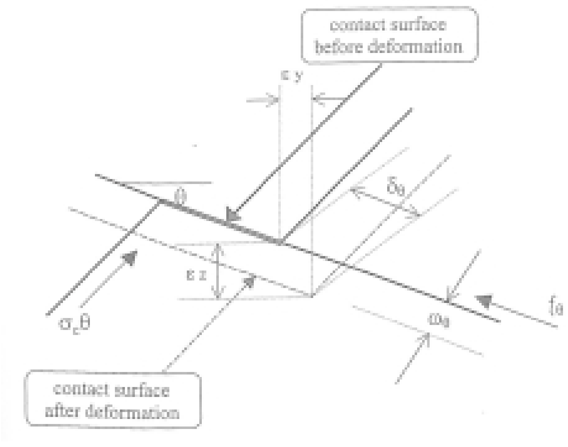

Fig 2. The 2-dimension Displacement Compatibility of a Contact at Contact Angle θ Showing Initial as well as Deformed Configurations of a Contact.

Contact Area

The factors affecting contact area of the particles are size, shape, gradation and re-arrangement of the particles. An important phenomenon is the increase of the contact area as the deformation progresses. By assuming that the contact area of a contact angle increases along with the amount of slip in that contact ($\delta\theta$). The contact area at a contact angle can be expressed as

$$A_{c\theta} = A_{co} + \int dA_{c\theta} \cdot \phi \tag{18}$$

where A_{co} is initial contact area (cm^2/m^3), and ϕ is a function to govern effect of particle size, shape, grading and re-arrangement on contact area. For crushed limestone coarse aggregate and river sand, ϕ equals to 1.4 and 1.1, respectively.

From the two-dimensional contact configuration in Fig 2, it can be assumed that the contact area of a constant angle θ increases along with the amount of slip (δ_θ) in that contact in a unit volume (m^3) so that a unit width (1 m) can be applied. As a unit of contact area is cm^2/m^3, a unit width in one meter is transformed to a hundred centimeters. A summation of increase of contact area can be derived from

$$\int dA_{c\theta} = 100 \cdot \delta_\theta \tag{19}$$

The initial contact area (A_{co}) can be assumed to be expressed by a non-linear function of the total surface area of aggregate (consider a unit volume of concrete). For crushed limestone coarse aggregate and river sand, the equation for A_{co} were found from the back analysis to be as in Eq.(20) and Eq.(21), respectively.

$$A_{co} = 16.48 \cdot \left(\frac{\zeta_g}{\zeta_{g,max}}\right) - 19.47 \cdot \left(\frac{\zeta_g}{\zeta_{g,max}}\right)^2 + 8 \cdot \left(\frac{\zeta_g}{\zeta_{g,max}}\right)^3 \tag{20}$$

$$A_{co} = 2.23 \cdot \left(\frac{\zeta_s}{\zeta_{s,max}}\right)^{0.77} \tag{21}$$

where ζ_G is the total surface area of coarse aggregate in a cubic meter of concrete (cm^2/m^3), and ζ_S is the total surface area of fine aggregate in a cubic meter of concrete. (cm^2/m^3). $\zeta_{G,max}$ is the total surface area of the densely compacted coarse aggregate in a cubic meter of bulk volume (cm^2/m^3), and $\zeta_{S,max}$ is the total surface area of the densely compacted fine aggregate in a cubic meter of bulk volume (cm^2/m^3). The surface area ratio ($\zeta_g/\zeta_{g,max}$, $\zeta_s/\zeta_{s,max}$) is equal to aggregate volume concentration ratio ($n_g/n_{g,max}$, $n_s/n_{s,max}$) that is defined as

$$\frac{\zeta_g}{\zeta_{g,max}} = \frac{n_g}{n_{g,max}} = \frac{(V_g/V_c)}{(V_{g,max}/V_c)} \tag{22}$$

$$\frac{\zeta_s}{\zeta_{s,max}} = \frac{n_s}{n_{s,max}} = \frac{(V_s/V_c)}{(V_{s,max}/V_c)} \tag{23}$$

where n_g is the coarse aggregate volume concentration, n_s is the fine aggregate volume concentration, $n_{g,max}$ is the aggregate volume concentration of the densely compacted coarse aggregate in a cubic meter of bulk volume, $n_{s,max}$ is the aggregate volume concentration of the densely compacted fine aggregate in a cubic meter of bulk volume, V_g is the volume of coarse aggregate (m^3), V_s is the volume of fine aggregate (m^3), V_c is the volume of concrete (m^3), $V_{g,max}$ is the densely compacted coarse aggregate volume in a cubic meter of bulk volume (m^3), and $V_{s,max}$ is the densely compacted fine aggregate volume in a cubic meter of bulk volume (m^3).

Then, the initial contact area in (A_{co}) in Eq.(20) and Eq.(21) can be modified as

$$A_{co} = 16.48 \cdot \left(\frac{n_g}{n_{g,max}}\right) - 19.47 \cdot \left(\frac{n_g}{n_{g,max}}\right)^2 + 8 \cdot \left(\frac{n_g}{n_{g,max}}\right)^3 \tag{24}$$

$$A_{co} = 2.23 \cdot \left(\frac{n_s}{n_{s,max}}\right)^{0.77} \tag{25}$$

Stiffness of Binary Mixtures of Aggregates

The above stiffness model was proposed for single materials (coarse and fine aggregates individually). However, aggregates in concrete are usually mixtures of coarse and fine aggregates. The stress of the mixture of aggregates is considered to be the combined results of stresses contributed by each single material, namely stress produced by coarse aggregates (σ_g) and stress produced by fine aggregates (σ_s). The stress contributed by coarse aggregates can be obtained from the summation of stresses from coarse aggregate - coarse aggregate interaction ($\sigma_{g\text{-}g}$) and fine aggregate - coarse aggregate interaction. The fine aggregate - coarse aggregate interaction stress can be obtained from the fine aggregate-fine aggregate interaction

($\sigma_{s\text{-}s}$) and the coarse aggregate volumetric ratio as

$$\sigma_g = \sigma_{g\text{-}g} + n_g \cdot \sigma_{s\text{-}s} \tag{26}$$

where σ_g is the stress produced by coarse aggregate (kgf/cm^2), $\sigma_{g\text{-}g}$ is the stress from coarse aggregate - coarse aggregate interaction (kgf/cm^2), $\sigma_{s\text{-}s}$ is the stress from fine aggregate - fine aggregate interaction (kgf/cm^2), and n_g is the coarse aggregate volume concentration.

In the same way, the stress contributed by fine aggregates can be obtained from the fine aggregate-fine aggregate interaction as

$$\sigma s = (1-n_g) \cdot \sigma_{s\text{-}s} \tag{27}$$

Then the total stress of the binary aggregate phase (σ_t) is calculated from

$$\sigma t = \sigma_g + \sigma_s = \sigma_{g\text{-}g} + \sigma_{s\text{-}s} \tag{28}$$

where $\sigma_{g\text{-}g}$ and $\sigma_{s\text{-}s}$ are obtained from the stiffness model of single material, so

$$\begin{aligned}\sigma_{t(z)} \;=\; & \int_0^{\pi/2} \Omega(\theta)\cdot(\sigma_{c\theta(g)}\cdot\cos\theta + f_{\theta(g)}\cdot\sin\theta)\cdot A_{c\theta(g)}\cdot d\theta \\ & + \int_0^{\pi/2} \Omega(\theta)\cdot(\sigma_{c\theta(s)}\cdot\cos\theta + f_{\theta(s)}\cdot\sin\theta)\cdot A_{c\theta(s)}\cdot d\theta\end{aligned} \tag{29}$$

$$\begin{aligned}\sigma_{t(y)} \;=\; & \int_0^{\pi/2} \Omega(\theta)\cdot(\sigma_{c\theta(g)}\cdot\sin\theta - f_{\theta(g)}\cdot\cos\theta)\cdot A_{c\theta(g)}\cdot d\theta \\ & + \int_0^{\pi/2} \Omega(\theta)\cdot(\sigma_{c\theta(s)}\cdot\sin\theta - f_{\theta(s)}\cdot\cos\theta)\cdot A_{c\theta(s)}\cdot d\theta\end{aligned} \tag{30}$$

where $A_{c\theta(g)}$ was defined in Eq.(18). $A_{co(g)}$ in this equation can be obtained from Eq.(24), while $A_{co(s)}$ can be obtained from

$$A_{co(s)} \;=\; 2.23 \cdot \left(\frac{n_s}{n_{s,max}\cdot(1-n_g)}\right)^{0.77} \tag{31}$$

The fine aggregate particles usually fill the voids among the coarse aggregates. When the mixture contains small amount of fine aggregates, they can fill the voids without disturbing the contacts among coarse aggregates. However, all fine aggregates cannot get inside the voids among coarse aggregates when there is a large amount of fine aggregate in the mixture. When the amount of fine aggregate is increased so that the particles of coarse aggregates are far apart from each other, the contact area of coarse aggregate will be reduced from this particle interference by fine aggregate as

$$A_{c\theta(g)} \;=\; (A_{co} + \int dA_\theta \cdot \phi)\cdot(1-\varphi) \tag{32}$$

where A_{co} of the coarse aggregate was defined in Eq.(24), and φ is the parameter to govern the effect of particle interference of fine aggregate on the contact area reduction of coarse aggregate which was derived to be

$$\varphi \;=\; \left(0.0056\cdot\frac{n_s}{n_{s,max}}\right)^{\left(0.15\cdot\left(\frac{n_g}{n_{g,max}}\right)^{0.73}\right)} \tag{33}$$

where n_s is fine aggregate volume concentration, and n_g is coarse aggregate volume concentration.

Then the stiffness of binary mixture of aggregate can be obtained from

$$E_{az} \;=\; \frac{\sigma_{t(z)}}{\varepsilon_z} \tag{34}$$

$$E_{ay} \;=\; \frac{\sigma_{t(y)}}{\varepsilon_y} \tag{35}$$

Reults and Discussion

To verify the versatility of the proposed aggregate stiffness model and the two-phase material concrete shrinkage model, autogenous shrinkage of mortar and concrete with different aggregate contents and ratio of fine to coarse aggregate was adopted from Deesawangnade.[6] Properties of the materials used in the experiments are given in Table 1. The tested mix proportions are listed in Table 2. Autogenous shrinkage strain of all specimens was obtained from test results with water to cement ratio equal to 0.30. The proposed model for stiffness of aggregates was utilized to compute the stiffness and compared with the results indirectly derived from back analysis of the tested shrinkage using the author's two-phase model for shrinkage of concrete. Fig 3 demonstrated the comparison results. It was found from both the tests and the model that stiffness increases with the increasing of volume concentration of aggregate.

Table 1. Physical properties of materials used in the test.

Material	Max size (mm)	Specific gravity (g/cm³)	Blaine fineness (cm²/g)	Absorption (%)	Void content (%)
Gravel	20	2.7	-	0.61	45.3
Sand	5	2.56	-	0.90	33.0
Cement Type 1	-	3.15	3467	-	-

Table 2. Mix proportion of autogenous shrinkage specimens of mortar, no-fine concrete and concrete.

Mix no	Designation	s/a (%)	w/c (%)	Cement (kg/m³)	Sand (kg/m³)	Gravel (kg/m³)	$\frac{n_a}{n_{a,max}}$	n_a
1	S-85	-	30	630	1457	-	0.85	0.570
2	S-70	-	30	792	1201	-	0.70	0.469
3	S-60	-	30	900	1029	-	0.60	0.402
4	S-50	-	30	1008	858	-	0.50	0.335
5	S-40	-	30	1116	686	-	0.40	0.268
6	G-100	-	30	661	-	1458	1.00	0.550
7	G-80	-	30	838	-	1188	0.80	0.440
8	G-60	-	30	1016	-	891	0.60	0.330
9	G-40	-	30	1194	-	594	0.40	0.220
10	SG25-70	25	30	805	320	961	0.70	0.481
11	SG50-70	50	30	723	699	699	0.70	0.532
12	SG75-70	75	30	768	980	327	0.70	0.504
13	SG25-50	25	30	1026	229	686	0.50	0.344
14	SG50-50	50	30	968	499	499	0.50	0.380
15	SG75-50	75	30	1000	700	233	0.50	0.360

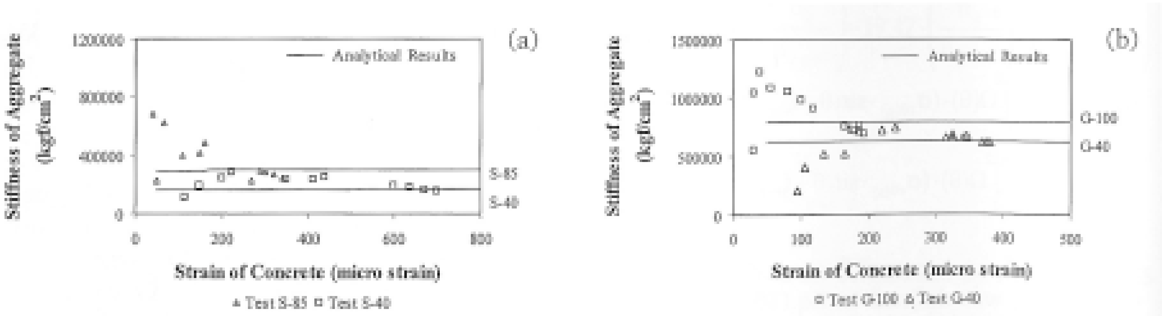

Fig 3. Test and analytical results of stiffness of (a) fine aggregate (b) coarse aggregate.

The reason is that the larger volume concentration has an effect to increase the stress produced by aggregate to obtain a greater stiffness. The comparison of stiffness of fine and coarse aggregate at the same volume concentration of aggregate is demonstrated in the Fig 4. It can be seen that the stiffness of fine aggregate is smaller than coarse aggregate at the same volume concentration because coarse aggregate produces higher stress due to larger contact area. The simulated results of aggregate stiffness of binary mixture are compared with the results from back analysis for mixtures with varied sand-aggregate ratio by weight (s/a) and volume concentration ratio of total aggregates ($n_a/n_{a,\,max}$) as shown in Fig 5. Higher aggregate stiffness of binary mixture is obtained when coarse aggregate content increases.

The two-phase model for simulating shrinkage of concrete taking into account the aggregate restraint stiffness was then utilized to compute the shrinkage of the tested specimens and compared with the test results. Fig 6 demonstrates the comparison. It was found from the test and analytical results that shrinkage decreases with the increasing of stiffness

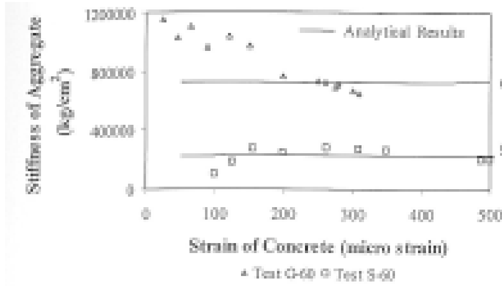

Fig 4. The comparison between stiffness of fine and coarse aggregates with similar n_a.

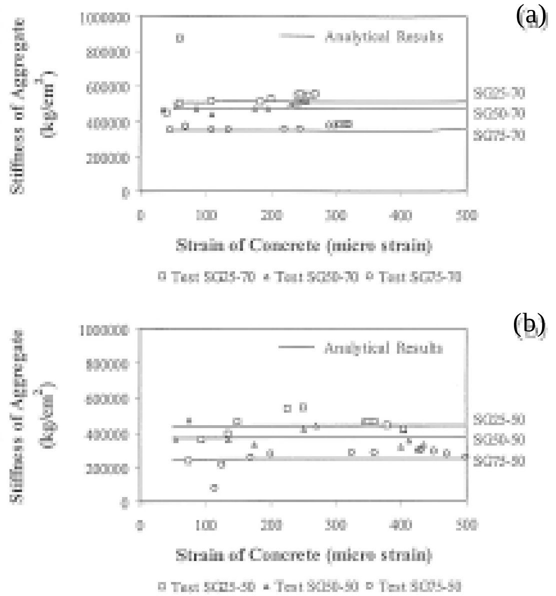

Fig 5. Test and analytical results of stiffness of binary mixture with (a) $n_a/n_{a,max}$ = 0.70 (b) $n_a/n_{a,max}$ = 0.50.

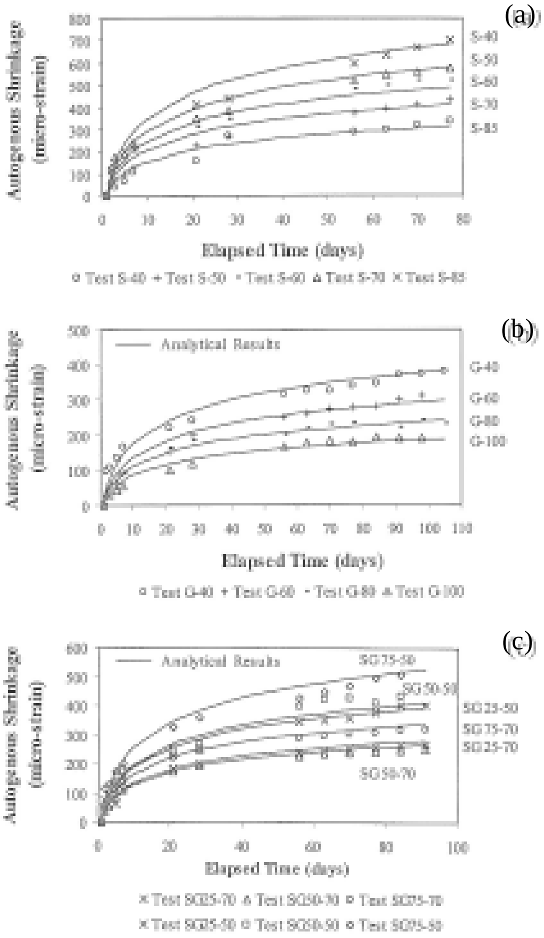

Fig 6. Test and analytic results of autogenous shrinkage of (a) mortar (b) no-fine concrete (c) concrete with binary mixture.

of aggregate and the increasing of volume concentration of total aggregates (n_a).

As from the comparison, the proposed model of effect of aggregate on shrinkage model is proved to be satisfactory for simulating the stiffness of aggregate and then the shrinkage of concrete as a two-phase material.

Conclusions

A mathematical model for simulating the shrinkage of concrete was derived based on concrete as a two-phase material. This model involved the stiffness, equilibrium condition of stress and strain compatibility of paste phase and aggregate phase. The shrinkage restraint of aggregate phase was expressed in terms of stiffness of aggregate. The model for simulating the stiffness of aggregate phase was derived based on the particle contact density concept and two-dimensional constitutive condition. Shrinkage tests were conducted on mortar and concrete specimens to verify the aggregate stiffness model for fine aggregate, coarse aggregate, and binary mixture of aggregate. It was found from the comparison between the back analysis stiffness from the two-phase material shrinkage model and analytical results from the proposed aggregate stiffness model that the model was effective for computing the restraint stiffness of fine aggregate, coarse aggregate, and combination of fine and coarse aggregate at different sand-aggregate ratio. It was found from both the tests and the model that aggregate stiffness increases with the increasing of volume concentration of aggregate. The stiffness of coarse aggregate is larger than the stiffness of fine aggregate at the same volume concentration. The shrinkage of concrete was found to decrease when the stiffness of aggregate increases and the volume concentration of total aggregates increases.

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