RESEARCH ARTICLES

J. Sci. Soc. Thailand, **4** (1978) 16-26

THE STATISTICAL DISTRIBUTION OF HOURLY SOLAR RADIATION AMOUNTS IN THAILAND

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(Received 27 December 1977)

Summary

Tables are given of the statistical distributions of the ratio of actual hourly solar radiation amounts to maximum possible amounts at Chiang Mai and Bangkok for six classes of daily total solar radiation and for eight $1\frac{1}{2}$ month periods of the year. The use of these tables to simulate the fluctuating sequence of hourly solar radiation amounts is discussed.

Introduction

Theoretical studies of the operation of a solar energy device often require a simulation of the fluctuating solar energy input throughout the day. Since the records of solar radiation in Thailand are incomplete, a study of the frequency distributions describing this complicated random process in the available data is necessary.

The observed diurnal variation of mean global solar radiation at Chiang Mai and Bangkok has been reported in this journal[1]. Hourly totals of global solar radiation between 7 h and 17 h apparent solar time averaged over eight $1\frac{1}{2}$ month periods of the year defined by standard solar declinations were given. The extent to which the hourly totals fluctuate about their mean value has also been determined, and the results expressed in terms of the percentiles of the distribution of hourly totals have been published[2], but these data are not sufficient to allow the construction of an adequate simulation model of the process.

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A more detailed model that appears at first to be suitable is the following. The daily total solar radiation amounts are divided into classes of width 100 cal $cm^{-2}d^{-1}$ and the hourly totals are divided into classes of width 10 cal $cm^{-2}h^{-1}$. Then for each $1\frac{1}{2}$ month period and for each class of daily totals the percentage frequency distributions among the classes of hourly totals are given for each hour of the day. Fig. 1 shows an example of the frequency distributions throughout the day in one $1\frac{1}{2}$ month period at Chiang Mai (obtained by a method to be explained later).

A disadvantage of this model is that a large number of percentage frequencies have to be recorded. The following considerations show that a more concise representation is possible. Fig. 1 shows that the pattern of distribution is dominated by the hourly change in the solar altitude. However, it is the variations in cloudiness that produce the fluctuations under investigation. Therefore a factor k was defined by the equation

$$k = Q/Q_m, \qquad (1)$$

where Q denotes the observed hourly total solar radiation and Q_m denotes the maximum possible hourly total under a clear sky during the same interval. Fig. 2 shows the distributions of k corresponding to the solar radiation distributions in Fig. 1. They are nearly the same throughout the whole day. Consequently, if the average of the distributions shown in Fig. 2 is used to represent the fluctuations in every hour interval only small errors will occur in the representation. The purpose of this paper is to report the average distributions of k that have been found in the hourly global solar radiation data from Chiang Mai and Bangkok during the years 1968 to 1972.

Reduction of the Data

In order to use the available data it is necessary to make a conversion to apparent solar time (AST) from zone mean time (ZMT), which is influenced by the equation of time and the difference in longitude between the observing station and the standard meridian 105°E.

The conversion of the data from hourly totals ZMT to hourly totals AST has been accomplished by the following method (see Fig. 3). First we note that both at Bangkok and at Chiang Mai the sun always crosses the meridian after 12 h ZMT, the delay ranging from 2 to 38 min. Suppose 12 h AST comes s min after 12 h ZMT, and let $t = 60 - s$. Let Q denote the reported total hourly solar radiation in the interval from n h ZMT to (n+1) h ZMT, where n is an integer. Let p denote the maximum possible total hourly solar radiation in the interval from (n−1) h AST to n h AST, and let q denote the maximum possible solar radiation in the interval from n h AST to (n+1) h AST. For this part of the calculation quantities proportional with sufficient accuracy to p and q are obtained by integrating $\cos(\pi t/2t_s)$ over the time interval with respect to the apparent solar time t counted from noon, where t_s is the apparent time of sunset. Then the quantity of radiation $psQ/(ps+qt)$ is assigned to the interval from (n−1) h AST to n h AST, and the quantity $qtQ/$

(ps+qt) is assigned to the interval from n h AST to (n+1) h AST. All the reported total hourly solar radiation amounts ZMT are divided among the hourly intervals in the same way. The above formulae ensure that both the delay in the sun crossing the meridian and also the diurnal variation of solar radiation intensity are properly taken into account. The method has, however, the embarrassing effect when s is large of smoothing out the very fluctuations it is intended to study. There seems to be no way of overcoming this difficulty. The conversion is necessary because large changes in the equation of time during individual 1½ month periods prevent one from combining unconverted data for different days in the same period. Fig. 4 shows an example of the conversion.

The calculation of Q_m in equation (1) was made with the help of the equation

$$\sin(A) = \cos(L)\cos(D)\cos(H) + \sin(L)\sin(D), \qquad (2)$$

where A is the solar altitude, L is latitude, D the solar declination, and H the hour angle of the sun, and also the empirical formula

$$G = 0.282p + 0.101p(p-1)/2! - 0.142p(p-1)\ (p-2)/3! + 0.103p\ldots (p-3)/4! - 0.082p\ldots(p-4)/5! + 0.061p\ldots(p-5)/6!, \qquad (3)$$

where G is an ideal value for the maximum possible solar radiation flux under clear skies in cal $cm^{-2}min^{-1}$, and $p = A/15^\circ$. Equation (3) is Newton's interpolation formula for 15° intervals in A representing the global solar radiation for a standard cloudless atmosphere[4]. The value of Q_m was obtained by integrating G over each one hour period using the trapezium rule and values of A, p and G computed at 10 min intervals.

The factors k were then calculated by equation (1), and the required percentage distributions were obtained as follows. For each 1½ month period, the daily totals of solar radiation were divided into six classes of width 100 cal $cm^{-2}d^{-1}$. Next, for all days having complete data in the same 1½ month period and the same daily solar radiation class the value of k was calculated for each hour and assigned to one of ten classes, each of width 0.1. The occurrences in each class were then counted and used to obtain separate percentage distributions for each hour of the day, as exemplified in Fig. 2.

Results and Discussion

Although as mentioned earlier the distribution of k remains nearly the same throughout the day for a specified 1½ month period and daily total solar radiation class, two types of variation were noticed in the results. One is the tendency of the mean value of k to be less in the afternoon than in the morning, especially in the wet season. This effect, which can be discerned in Fig. 2, is generally small, as shown in a previous paper[5]. The other is the tendency of k to be less near sunrise and sunset than around midday. (This happens not to be prominent in Fig. 2.) It is doubtless caused by the fact that layers of cloud have more obscuring effect on oblique rays of sunlight near sunrise and sunset than on the nearly vertical rays around midday[6].

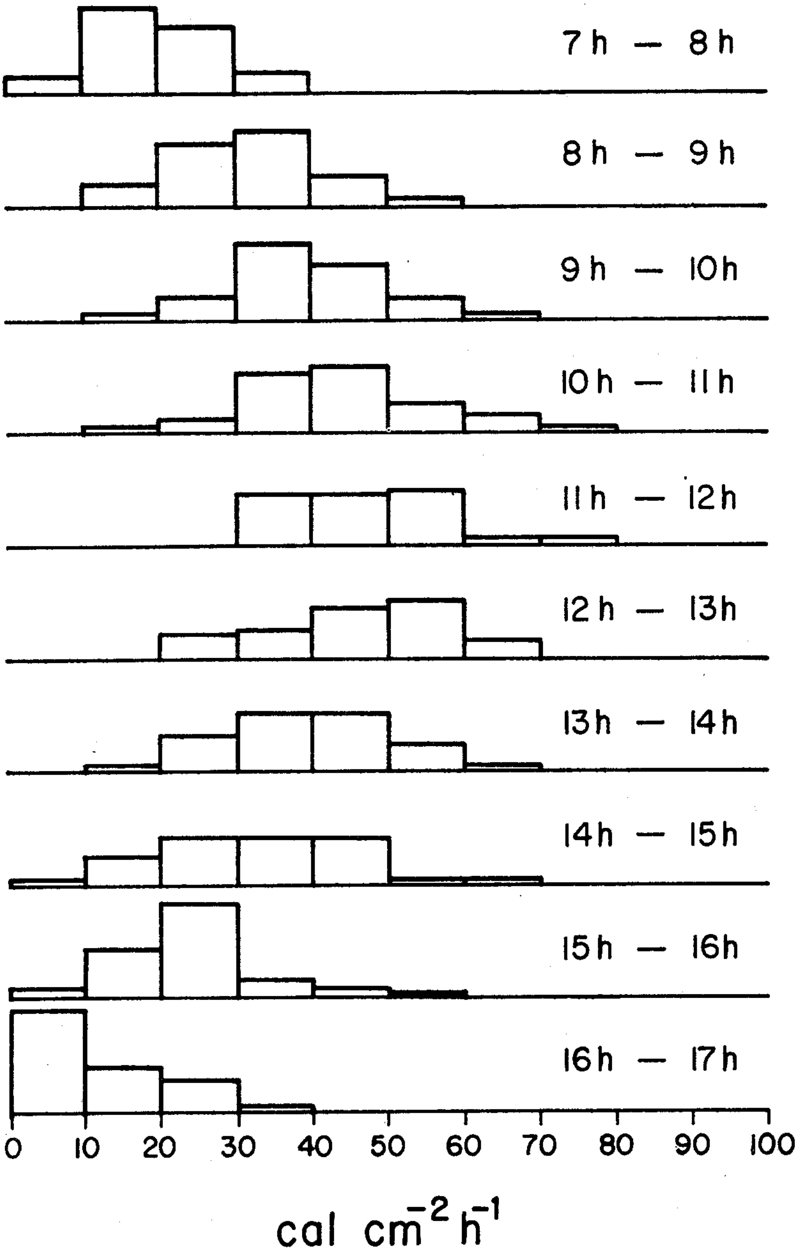

Fig. 1. Relative frequency histograms of hourly total solar radiation at Chiang Mai for days with daily total solar radiation from 300 to 400 cal cm^{-2}d^{-1} in the period 29 May to 15 July.

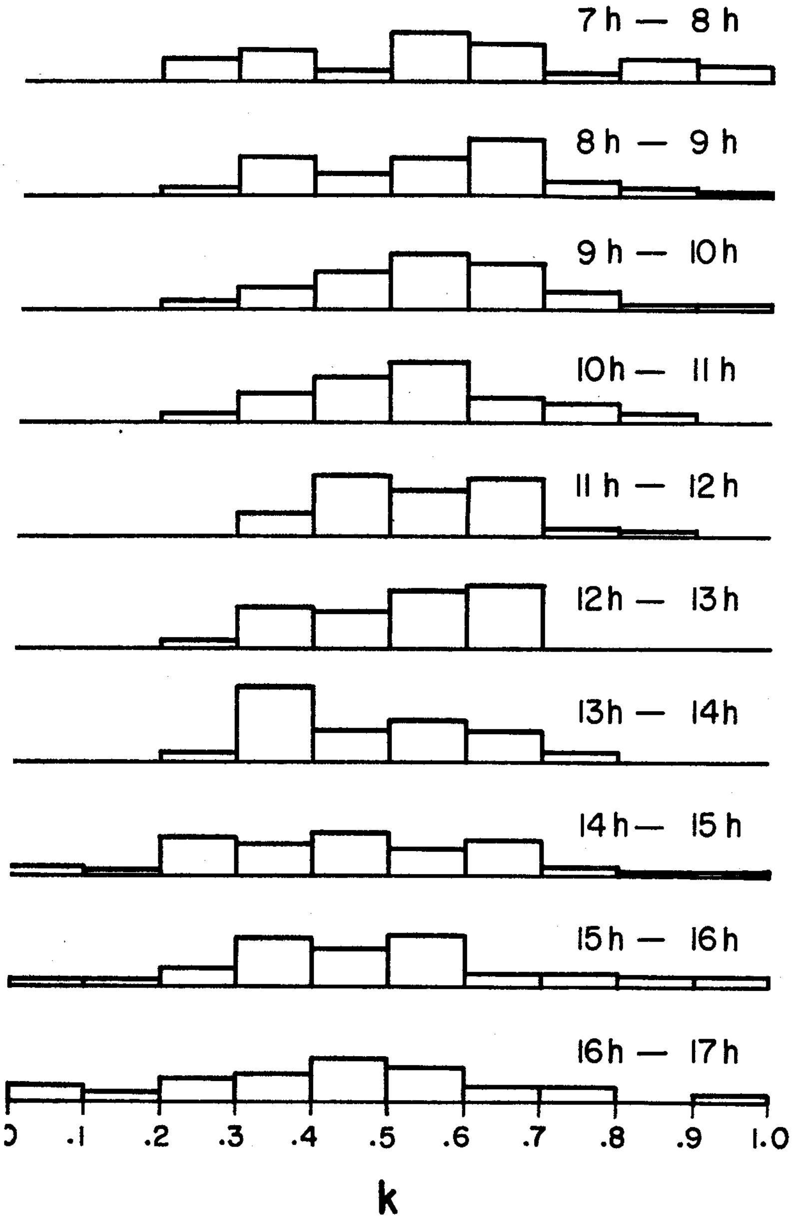

Fig. 2. Relative frequency histograms of values of k at Chiang Mai for days with daily total solar radiation from 300 to 400 cal $cm^{-2}d^{-1}$ in the period 29 May to 15 July.

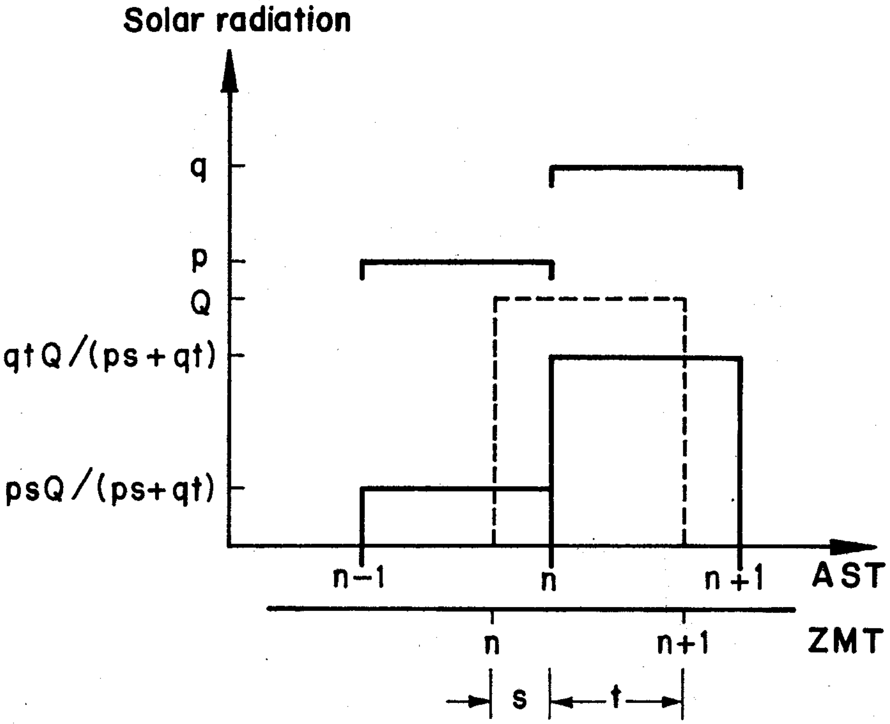

Fig. 3. Diagram to illustrate the conversion of hourly totals of solar radiation ZMT to hourly totals AST (see text).

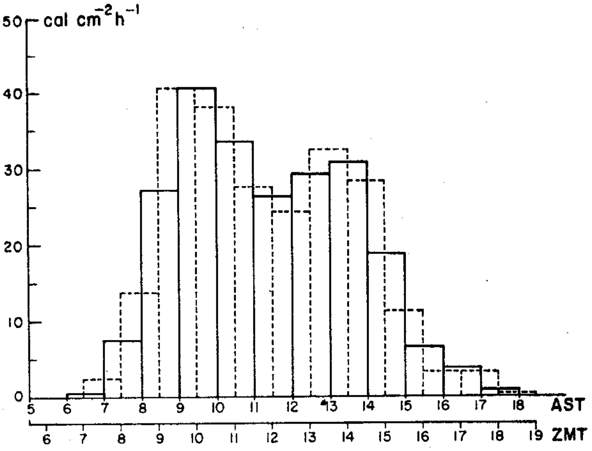

Fig. 4. Hourly totals of global solar radiation at Bangkok on 13 Feb 1970. Dashed lines: observed totals, full lines: converted totals. s = 32.5 min.

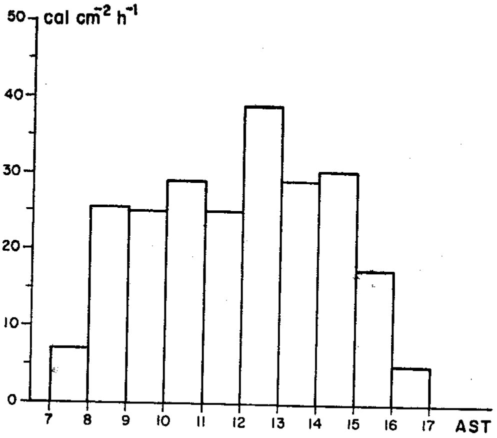

Fig. 5. Simulated hourly totals of global solar radiation at Bangkok for the period 14 Jan to 26 Feb and total solar radiation class 200 to 300 cal cm^{-2}d^{-1}.

TABLE I: PERCENTAGE DISTRIBUTIONS OF k AT CHIANG MAI

Total daily solar radiation (cal cm^{-2}d^{-1})	Values of k									
	0 to .1	.1 to .2	.2 to .3	.3 to .4	.4 to .5	.5 to .6	.6 to .7	.7 to .8	.8 to .9	.9 to 1.0
14 Jan–26 Feb										
over 600	0	0	0	0	0	0	0	0	0	0
500–600	0	0	0	0	0	0	0	10	74	16
400–500	0	0	0	0	0	1	6	25	65	3
300–400	0	0	0	2	5	16	35	36	6	0
200–300	0	0	6	17	31	19	24	3	0	0
under 200	0	0	0	0	0	0	0	0	0	0
27 Feb–12 Apr										
over 600	0	0	0	0	0	0	0	4	11	85
500–600	0	0	0	0	0	1	6	22	55	16
400–500	0	0	0	0	1	7	27	50	15	0
300–400	0	2	4	3	15	30	31	14	1	0
200–300	2	14	14	29	17	15	3	6	0	0
under 200	21	49	30	0	0	0	0	0	0	0
13 Apr–28 May										
over 600	0	0	0	0	0	0	0	8	34	58
500–600	0	0	0	1	3	6	9	12	47	22
400–500	0	2	3	4	8	10	18	24	25	6
300–400	0	4	6	8	24	23	19	11	5	0
200–300	0	7	29	34	11	11	8	0	0	0
under 200	0	0	0	0	0	0	0	0	0	0
29 May–15 Jul										
over 600	0	0	0	0	0	0	2	5	19	74
500–600	0	0	0	0	1	4	16	24	29	26
400–500	0	0	2	3	8	18	25	24	15	5
300–400	1	0	5	19	21	25	21	5	2	1
200–300	0	10	16	37	23	11	3	0	0	0
under 200	14	18	43	10	15	0	0	0	0	0
16 Jul–31 Aug										
over 600	0	0	0	0	1	1	0	8	23	67
500–600	0	0	0	0	3	7	13	21	24	32
400–500	0	0	2	3	12	18	24	23	13	5
300–400	0	1	5	12	26	29	17	7	3	0
200–300	1	8	17	32	27	10	3	2	0	0
under 200	9	29	39	18	3	2	0	0	0	0
1 Sep–15 Oct										
over 600	0	0	0	0	0	0	0	0	26	74
500–600	0	0	0	0	1	2	6	14	34	43
400–500	0	0	1	3	7	11	21	25	20	12
300–400	0	2	6	11	17	22	24	12	4	2
200–300	2	10	13	21	23	17	11	2	1	0
under 200	18	13	31	33	4	1	0	0	0	0
16 Oct–29 Nov										
over 600	0	0	0	0	0	0	0	0	0	0
500–600	0	0	0	0	0	0	3	6	69	22
400–500	0	0	0	1	1	3	9	14	57	15
300–400	0	1	5	8	11	16	25	19	11	4
200–300	1	2	10	21	25	24	10	4	2	1
under 200	4	19	27	39	11	0	0	0	0	0
30 Nov–13 Jan										
over 600	0	0	0	0	0	0	0	0	0	0
500–600	0	0	0	0	0	0	0	0	0	0
400–500	0	0	0	0	0	1	6	24	64	5
300–400	0	0	1	3	5	14	17	34	25	1
200–300	0	5	5	17	25	30	10	4	4	0
under 200	0	37	30	19	8	4	1	0	1	0

TABLE II: PERCENTAGE DISTRIBUTIONS OF k AT BANGKOK

Total daily solar radiation (cal cm^{-2}d^{-1})	Values of k									
	0 to .1	.1 to .1	.2 to .3	.3 to .4	.4 to .5	.5 to .6	.6 to .7	.7 to .8	.8 to .9	.9 to 1.0
14 Jan–26 Feb										
over 600	0	0	0	0	0	0	0	0	0	0
500–600	0	0	0	0	0	0	2	4	42	52
400–500	0	0	0	0	0	1	5	19	46	29
300–400	0	0	1	2	8	14	27	27	16	5
200–300	0	1	7	14	41	14	14	8	1	0
under 200	3	28	33	34	2	0	0	0	0	0
27 Feb–12 Apr										
over 600	0	0	0	0	0	0	0	0	0	100
500–600	0	0	0	0	0	0	6	22	56	16
400–500	0	0	0	1	2	5	20	38	32	2
300–400	0	4	4	7	16	17	37	12	3	0
200–300	1	13	17	21	14	10	17	7	0	0
under 200	7	62	31	0	0	0	0	0	0	0
13 Apr–28 May										
over 600	0	0	0	0	0	0	0	7	58	35
500–600	0	0	0	0	0	1	11	24	58	6
400–500	0	0	1	4	6	16	32	24	14	3
300–400	0	4	7	17	20	23	19	9	1	0
300–300	5	20	17	17	15	12	11	3	0	0
under 200	27	24	15	18	16	0	0	0	0	0
29 May–15 Jul										
over 600	0	0	0	0	0	0	0	0	0	0
500–600	0	0	0	0	2	3	15	39	36	5
400–500	0	0	1	3	7	22	25	31	10	1
300–400	0	2	6	11	25	30	21	5	0	0
200–300	1	6	23	29	28	7	5	1	0	0
under 200	26	25	25	18	2	2	0	2	0	0
16 Jul–31 Aug										
over 600	0	0	0	0	0	0	9	17	20	54
500–600	0	0	0	0	1	5	17	25	34	18
400–500	0	1	1	2	8	20	36	21	10	1
300–400	0	1	5	12	25	27	18	9	3	0
200–300	1	5	20	36	22	11	4	1	0	0
under 200	5	17	55	9	0	11	3	0	0	0
1 Sep–15 Oct										
over 600	0	0	0	0	0	0	0	0	0	0
500–600	0	0	0	0	1	1	5	13	30	50
400–500	0	0	0	2	7	10	24	25	20	12
300–400	0	4	4	10	20	21	17	14	7	3
200–300	6	9	21	16	18	16	9	2	2	1
under 200	29	31	21	12	2	3	1	1	0	0
16 Oct–29 Nov										
over 600	0	0	0	0	0	0	0	0	0	0
500–600	0	0	0	0	0	0	1	3	2	94
400–500	0	0	0	0	1	3	8	14	22	52
300–400	0	0	2	5	8	20	19	20	17	9
200–300	3	4	10	16	24	20	14	6	2	1
under 200	4	24	30	24	16	2	0	0	0	0
30 Nov–13 Jan										
over 600	0	0	0	0	0	0	0	0	0	0
500–600	0	0	0	0	0	0	0	0	0	0
400–500	0	0	0	0	0	0	1	5	18	76
300–400	0	0	1	2	4	10	15	26	29	13
200–300	0	1	4	14	21	22	16	13	8	1
under 200	0	4	37	14	41	4	0	0	0	0

The major contribution to the total daily solar radiation is that received during the middle of the day. Therefore, for practical purposes in the utilization of solar energy it is the distribution of k around midday that is the most important. Accordingly, the percentage distributions of k quoted in this paper are weighted averages of the distributions for the eight intervals from 8 h to 16 h. The weights used are proportional to the cosine of the hour angle of the sun at the mid-points of the intervals, which makes them roughly proportional to the average amount of solar radiation received in the interval.

The results are shown in Tables I and II. A suggestion as to the manner in which they might be used to simulate the sequence of hourly solar radiation amounts is as follows. First one simulates the series of daily totals of solar radiation using previously published tables of probabilities for the distributions of daily totals of global solar radiation and the transitions between solar radiation classes on successive days[1,2,5]. Then for each day in the series one uses the percentage frequency distribution of the factors k (from Tables I and II) corresponding to the appropriate daily total class together with equations (1), (2) and (3) to simulate the hourly radiation amounts. This is a second order (Markov) model of the daily totals, but a first order (purely random) model of the hourly totals. It takes into account the persistence of weather conditions from day to day governing the daily totals but not the persistence of weather conditions from hour to hour that might govern the hourly totals. However, incorporating the hourly persistence of weather conditions into the model would make it so unwieldy as to be impractical.

A number of simulations of this type were made under the conditions in the case of the actual data in Fig. 2, namely the period 14 Jan. to 26 Feb. and total solar radiation class 200 to 300 cal $cm^{-2}d^{-1}$. They all had an appearance similar in general character to the actual data. An example is given in Fig. 5.

Acknowledgements

We are indebted to the Meteorological Department, Ministry of Communications, for kindly supplying data on solar radiation, and Md. Maminul Huq is indebted to the Belgian Government for financial assistance during his course of study in the Asian Institute of Technology.

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บทคัดย่อ

ได้แสดงตัวเลขจากผลของสถิติของการจำแนกการเทียบอัตราส่วนของปริมาณพลังแสงอาทิตย์ที่ปรากฏจริงโดยรายชั่วโมงกับปริมาณของพลังแสงอาทิตย์ในขอบเขตที่จะเป็นไปได้ในอัตราสูงสุดทั้งที่เชียงใหม่และที่กรุงเทพฯ การวัดผลของพลังแสงอาทิตย์เป็นรายวันโดยเบ็ดเสร็จนั้น จัดออกเป็นหกกลุ่มด้วยกัน โดยแบ่งการวัดเป็นแปดระยะ แต่ละระยะใช้เวลา $1\frac{1}{2}$ เดือน อนึ่งสำหรับกรณีที่จะนำตัวเลขในตารางดังกล่าวมาใช้ในการเปรียบเทียบกับผลสำรวจของความไม่คงที่ของพลังแสงอาทิตย์รายชั่วโมง ที่กระทำต่อเนื่องกันนั้น ได้บรรยายไว้อย่างละเอียดด้วย